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Energy and Magnitude: A Historical Perspective

EMILE A. OKAL[1]

Abstract—We present a detailed historical review of early attempts to quantify seismic sources through a measure of the energy radiated into seismic waves, in connection with the parallel development of the concept of magnitude. In particular, we explore the derivation of the widely quoted "Gutenberg–Richter energy–magnitude relationship"

$$\log_{10} E = 1.5 M_s + 11.8 \qquad (1)$$

(*E* in ergs), and especially the origin of the value 1.5 for the slope. By examining all of the relevant papers by Gutenberg and Richter, we note that estimates of this slope kept decreasing for more than 20 years before Gutenberg's sudden death, and that the value 1.5 was obtained through the complex computation of an estimate of the energy flux above the hypocenter, based on a number of assumptions and models lacking robustness in the context of modern seismological theory. We emphasize that the scaling laws underlying this derivation, as well as previous relations with generally higher values of the slope, suffer violations by several classes of earthquakes, and we stress the significant scientific value of reporting radiated seismic energy independently of seismic moment (or of reporting several types of magnitude), in order to fully document the rich diversity of seismic sources.

Key words: Radiated seismic energy, earthquake magnitudes, historical seismicity, seismic scaling laws.

1. Introduction

This paper presents a historical review of the measurement of the energy of earthquakes, in the framework of the parallel development of the concept of magnitude. In particular, we seek to understand why the classical formula

$$\log_{10} E = 1.5 M_s + 11.8 \qquad (1)$$

[1] Department of Earth and Planetary Sciences, Northwestern University, Evanston, IL 60208, USA. E-mail: emile@earth.northwestern.edu

referred to as "Gutenberg [and Richter]'s energy–magnitude relation" features a slope of 1.5 which is not predicted a priori by simple physical arguments. We will use Gutenberg and Richter's (1956a) notation, *Q* [their Eq. (16) p. 133], for the slope of $\log_{10} E$ versus magnitude [1.5 in (1)].

We are motivated by the fact that Eq. (1) is to be found nowhere in this exact form in any of the traditional references in its support, which incidentally were most probably copied from one referring publication to the next. They consist of Gutenberg and Richter (1954) (*Seismicity of the Earth*), Gutenberg (1956) [the reference given by Kanamori (1977) in his paper introducing the concept of the "moment magnitude" M_w], and Gutenberg and Richter (1956b). For example, Eq. (1) is not spelt out anywhere in Gutenberg (1956), although it can be obtained by combining the actual formula proposed for *E* [his Eq. (3) p. 3]

$$\log_{10} E = 2.4 m + 5.8 \qquad (2)$$

with the relationship between the "unified magnitude" *m* (Gutenberg's own quotes) and the surface-wave magnitude M_s [Eq. (1) p. 3 of Gutenberg (1956)]:

$$m = 0.63 M_s + 2.5, \qquad (3)$$

neither slope (2.4 or 0.63) having a simple physical justification. The same combination is also given by Richter (1958, pp. 365–366), even though he proposes the unexplained constant 11.4 instead of 11.8 in (1), a difference which may appear trivial, but still involves a ratio of 2.5. It is also given in the caption of the nomogram on Fig. 2 of Gutenberg and Richter (1956b), which does provide separate derivations of (2) and (3). As for Gutenberg and Richter (1954) (the third edition, generally regarded as definitive, of

Seismicity of the Earth), the only mention of energy is found in its Introduction (p. 10)

"In this book, we have assumed for radiated energy the partly empirical equation

$$\log_{10} E = 12 + 1.8M. \tag{4}$$

This seems to give too great energy. At present (1953), the following form is preferred:

$$\log_{10} E = 11 + 1.6M''. \tag{5}$$

While Eq. (4), with $Q = 1.8$, was derived from Gutenberg and Richter (1942), Eq. (5), with $Q = 1.6$, was apparently never formally published or analytically explained.

The fact that none of the three key references to "Gutenberg and Richter's energy–magnitude relation" actually spells it out warrants some research into the origin of the formula, from both historical and theoretical standpoints. In order to shed some light on the origin of (1), and to recast it within modern seismic source theory, this paper explores the development of the concept of earthquake energy and of its measurement, notably in the framework of the introduction of magnitude by Richter (1935). In particular, we examine all of Gutenberg's papers on the subject, using the compilation of his bibliography available from his obituary (Richter 1962).

2. The Modern Context and the Apparent "Energy Paradox"

Understanding the evolution of the concept of magnitude and the attempts to relate it to seismic energy must be based on our present command of seismic source theory. In this respect, this section attempts to provide a modern theoretical forecast of a possible relation between magnitude and energy. We base our discussion on the concept of *double couple* **M** introduced by Vvedenskaya (1956), and later Knopoff and Gilbert (1959) as the system of forces representing a seismic dislocation, its scalar value being the seismic moment M_0 of the earthquake.

Note that we consider here, as the "energy" of an earthquake only the release of elastic energy stored during the interseismic deformation of the Earth, and not the changes in gravitational and rotational kinetic energy resulting from the redistribution of mass during the earthquake, which may be several orders of magnitude larger (Dahlen 1977).

2.1. The Energy Paradox

We first recall that *magnitude* was introduced by Richter (1935) as a measure of the logarithm of the amplitude of the seismic trace recorded by a torsion instrument at a distance of 100 km, and thus essentially of the ground motion generated by the earthquake. In the absence of source finiteness effects, and given the linearity of the equations of mechanics governing the Earth's response [traceable all the way to Newton's (1687) "$\mathbf{f} = m\,\mathbf{a}$"], that ground motion, A in the notation of most of Gutenberg's papers, should be proportional to M_0, and hence any magnitude M should grow like $\log_{10} M_0$. This is indeed what is predicted theoretically and observed empirically, for example for the surface-wave magnitude M_s below about six (Geller 1976; Ekström and Dziewoński 1988; Okal 1989).

In most early contributions, it was generally assumed that the energy of a seismic source could be computed from the kinetic energy of the ground motion imparted to the Earth by the passage of a seismic wave, which would be expected to grow as the square of the amplitude of ground motion. Since the concept of magnitude measures the logarithm of the latter, this leads naturally to $Q = 2$, as featured by earlier versions of Eq. (1) (Gutenberg and Richter 1936).

By contrast, using the model of a double-couple **M**, the seismic energy E released by the source is simply its scalar product with the strain ε released during the earthquake. The absolute value of the strain should be a characteristic of the rock fracturing during the earthquake, and as such an invariant in the problem, so that E should be proportional to M_0. Again, in the absence of source finiteness effects, the linearity between seismic source and ground motion ("$\mathbf{f} = m\,\mathbf{a}$") will then result in E being directly proportional to ground motion, and hence in a slope $Q = 1$ in Eq. (1).

We thus reach a paradox, in that the two above arguments predict contradictory values of Q. The highly quoted Gutenberg and Richter relationship (1),

which uses the intermediate value $Q = 1.5$, may appear as a somewhat acceptable compromise, but satisfies neither interpretation. Thus, it deserves full understanding and discussion.

2.2. A Modern Approach

The origin of this apparent, and well-known, "energy paradox" can be traced to at least three effects:

(i) Most importantly, the proportionality of energy to the square of displacement holds only for a monochromatic harmonic oscillator (with the additional assumption of a frequency not varying with size), while the spectrum of seismic ground motion following an earthquake is distributed over a wide range of frequencies, and thus the resulting time-domain amplitude at any given point (which is what is measured by a magnitude scale) is a complex function of its various spectral components;

(ii) Because of source finiteness ("a large earthquake takes time and space to occur"), destructive interference between individual elements of the source causes ground motion amplitudes measured at any given period to grow with moment slower than linearly, and eventually to saturate for large earthquakes (Geller 1976);

(iii) Ground motion can be measured only through the use of instruments acting as filters; while some of them could in principle be so narrow as to give the seismogram the appearance of a monochromatic oscillator, justifying assumption (i), the inescapable fact remains that most of the energy of the source would then be hidden outside the bandwidth recorded by the instrument. In parallel, ground motion can be measured only at some distance from the source, and anelastic attenuation over the corresponding path will similarly affect the spectrum of the recorded seismogram.

In modern days, the effect of (iii) is vastly reduced by the availability of broadband instrumentation. Using modern theory, it is possible to offer quantitative

models of the concept of finiteness (ii), as first described by Ben-Menahem (1961), and, when integrating it over frequency [which takes care of (i)], to reconcile quantitatively the paradox exposed above.

In practice, seismic magnitudes have been, and still are, measured either on body waves, or on surface waves (exceptionally on normal modes). As discussed by Vassiliou and Kanamori (1982), the energy radiated in P and S wavetrains can be written as

$$E^{\text{Body}} = F^{\text{B}} \cdot \frac{M_0^2}{t_0^3} \qquad (6)$$

where t_0 represents the total *duration* of the source (the inverse of a *corner frequency*), which under seismic scaling laws (Aki 1967) is expected to grow like $M_0^{1/3}$, and F^{B} is a combination of structural parameters (density, seismic velocities) and of the ratio x of rise time to rupture time, which are expected to remain invariant under seismic scaling laws. As a result, E/M_0 is also expected to remain constant, its logarithm being estimated at -4.33 by Vassiliou and Kanamori (1982), and -4.90 by Newman and Okal (1998). Extensive datasets compiled by Choy and Boatwright (1995) and Newman and Okal (1998) have upheld this invariance of $\log_{10}(E/M_0)$ with average values for shallow earthquakes of -4.80 and -4.98, respectively, a result later upheld even for microscopic sources by Ide and Beroza (2001).

In the case of surface waves, we have shown (Okal 2003) that the energy of a Rayleigh wave can be similarly expressed as

$$E^{\text{Rayleigh}} = F^{\text{R}} \cdot \omega_{\text{c}}^3 M_0^2, \qquad (7)$$

where ω_{c} is a corner frequency, not necessarily equal to its body wave counterpart, but still expected to behave as $M_0^{-1/3}$ [see Eq. (A8) of Okal (2003)], and F^{R} is a combination of structural parameters and Rayleigh group and phase velocities, which can be taken as constant. While we argue in Okal (2003) that the energy carried by Rayleigh waves represents only a small fraction (less than 10 %) of the total energy released by the dislocation, the combined result from (6) and (7) is that energy, when properly measured

across the full spectrum of the seismic field, does grow linearly with the seismic moment M_0.

As for the growth of magnitudes, Geller (1976) has used the concept of source finiteness heralded by Ben-Menahem (1961) to explain how any magnitude measured at a constant period T starts by being proportional to $\log_{10} M_0$ for small earthquakes, and then grows slower with moment, as the inverse of the corner frequencies characteristic of fault length, rise time, and fault width become successively comparable to, or longer than, the reference period T. Under Geller's (1976) model, the slope of M versus $\log_{10} M_0$ should decrease to 2/3, then 1/3, and finally 0, as any magnitude measured at a constant period reaches an eventual saturation. The latter is predicted and observed around 8.2 for the 20-s surface-wave magnitude M_s, and predicted around 6.0 for m_b, if consistently measured at 1 s (occasional values beyond this theoretical maximum would reflect measurements taken at periods significantly longer than the 1-s standard). Figure 1 plots this behavior (m_b as a long-dashed blue curve, M_s in short-dotted red), as summarized by the last set of (unnumbered) equations on p. 1520 of Geller (1976). Because of the straight proportionality between E and M_0, the vertical axis also represents $\log_{10} E$, except for an additive constant.

The conclusion of these theoretical remarks is that Q is expected to grow with earthquake size, from its unperturbed value of 1 in the domain of small sources unaffected by finiteness, to 1.5 under moderate finiteness, and to a conceptually infinite value when M has fully saturated. Note that these conclusions will hold experimentally only under two conditions: (1) that the energy should be computed (either from body or surface waves) using an integration over the full wave packet, in either the time or frequency domain, these two approaches being equivalent under Parseval's theorem; and (2) that magnitudes for events of all sizes should be computed using the same algorithms, most importantly at constant periods.

3. The Quest for Earthquake Energy: A Timeline

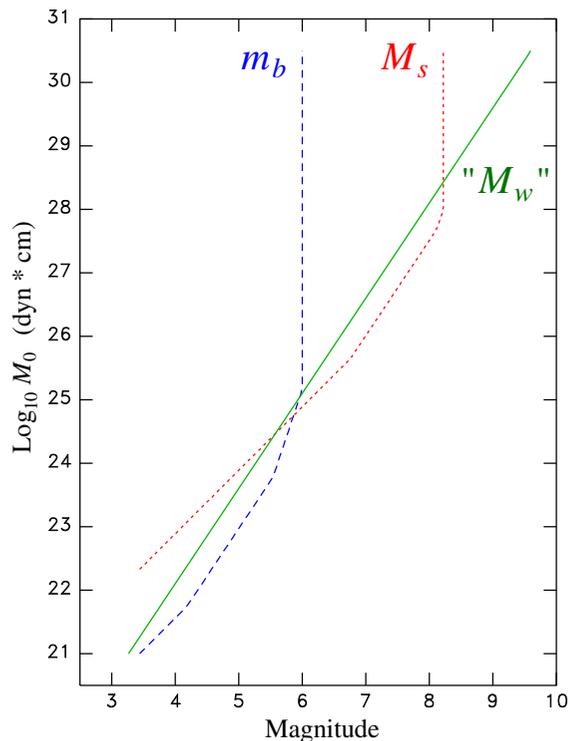In this general framework, we present here a timeline of the development of measurements of

Figure 1
Variation of body-wave magnitude m_b (long dashes, blue) and surface-wave magnitude M_s (short dots, red) with seismic moment M_0, predicted theoretically from Eqs. (16) and (17) (Geller 1976). Superimposed in solid green is the relationship (18), casting values of M_0 into the scale M_w (Kanamori 1977)

earthquake energy, and of the refinement of the concept of magnitude. The discussion of the critical papers by Gutenberg and Richter which underlie Eq. (1) will be reserved for Sect. 4 and further detailed in the Appendix; contributions from other authors will be discussed here.

1. To our knowledge, the first attempt at quantifying the energy released by an earthquake goes back to Mendenhall (1888), who proposed a value of 3.3×10^{21} erg (2.4×10^{14} ft lbs) for the 1886 Charleston earthquake. This figure is absolutely remarkable, given that modern estimates of the moment of the event are around $(1-10) \times 10^{26}$ dyn cm (Johnston 1996; Bakum and Hopper 2004), which would suggest an energy of about $(1.2-12) \times 10^{21}$ erg, according to global scaling laws (Choy and Boatwright 1995; Newman and Okal 1998). Mendenhall's (1888) calculation was based on Lord Kelvin's

description of the energy of a "cubic mile" of vibrating matter moving with the wave front (Thomson 1855), a concept nowadays expressed as an *energy flux* (Wu 1966; Boatwright and Choy 1986). However, Mendenhall uses grossly inadequate estimates of particle velocities (on the order of 15 cm/s), and of the cross section and "thickness" of the wavefront, taken as 10^4 square miles and 1 mile, respectively. Thus the surprising accuracy of his result stems from the fortuitous compensation of fatally erroneous assumptions.

2. By contrast, the first calculation of the energy released by an earthquake whose methodology would be upheld under present standards goes back to Reid (1910). In his comprehensive report on the 1906 San Francisco earthquake, Reid proposed a value of 1.75×10^{24} erg, based on an estimate of the forces necessary to offset the strain accumulated around the fault. This number is about 10 times too large given generally accepted values of the event's seismic moment (Wald et al. 1993), but its computation is nevertheless remarkable.

A year later, Reid embarked on measuring the energy of 12 additional earthquakes from an analysis of the areas of their Rossi–Forel intensity III isoseismals, scaled to that of the 1906 San Francisco earthquake, an idea originally found in Milne (1898). Reid does comment on the apparent scatter of his results, and in a truly visionary discussion, identifies probable sources of errors which would be described in modern terms as source radiation patterns, departure from earthquake scaling laws, and even the preferential attenuation of shear waves by the asthenosphere (Reid 1912).

3. The next reference to the computation of an earthquake's energy seems to be the work of Golitsyn (1915) on the Sarez earthquake of 18 February 1911[1] in present-day Tajikistan, which was later assigned a magnitude of $7\frac{3}{4}$ by Gutenberg and Richter (1954). Golitsyn's (1915)

computation is important, because it is the first one making use of a recorded seismogram. It was then revised and discussed, first by Klotz (1915) and later by Jeffreys (1923, 1929), and it became part of the small original dataset used by Gutenberg and Richter (1936) in their first attempt to relate magnitude to energy [see (6) in this list]. An extensive study of the Sarez earthquake was recently published by Ambraseys and Bilham (2012), who recomputed a surface-wave magnitude $M_s = 7.7$, but did not perform any waveform analysis. These numbers would suggest a moment of about 5×10^{27} dyn cm, under the assumption that the event follows scaling laws. We were able to obtain an independent, modern, estimate of the moment of the earthquake by applying the M_m algorithm (Okal and Talandier 1989, 1990) to original Love and Rayleigh wavetrains recorded at De Bilt and Uppsala, yielding an average value of $M_0 = 3 \times 10^{27}$ dyn cm. As detailed by Ambraseys and Bilham (2012), the earthquake was accompanied by a catastrophic landslide, later surveyed by the Imperial Russian Army at an estimated volume of 2.5–3.5 km^3, which dammed the Murgab River, creating Sarez Lake, a 17-km^3 reservoir (Shpil'ko 1914, 1915); incidentally, the potential failure of the resulting natural Usoi Dam remains to this day a significant hazard in the region (Lim et al. 1997; Schuster and Alford 2004), especially in view of recent large-scale seismic activity in its neighborhood (Elliott et al. 2017; Negmatullayev et al. 2018).

Golitsyn's purpose in computing the energy of the seismic waves was to address the question of the causality of the landslide, i.e., was it a result of the earthquake or the opposite, the argument being that, if the slide was caused by the earthquake, it should have only a fraction of the seismic energy released. Golitsyn's (1915) work constituted a significant improvement on Mendenhall's (1888) in that he used actual seismograms (in this case Rayleigh waves at Pulkovo) to compute an energy flux, which he then integrated over the observed duration of the wavetrain, and, mistakenly as noticed by Jeffreys

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[1] In the Gregorian calendar ("new style"). The date is 5 February in the Julian calendar ("old style") used in the Russian Empire at the time.

(1923), over the lower focal hemisphere, to derive the energy released at the source. Golitsyn (1915) obtained a value of 4.3×10^{23} erg, comparable to his estimate of $(2–6) \times 10^{23}$ erg for the energy of the rockslide, and concluded that the slide was "not the consequence, but the cause of the earthquake." Once again, and remarkably, his value of the seismic energy is not unreasonable, being only ten times larger than expected under the assumption of scaling laws (Boatwright and Choy 1986; Newman and Okal 1998) applied to our estimate of the seismic moment (3×10^{27} dyn cm), but as we will show, this remains a coincidence, resulting from a number of compensating errors.

In what amounts to a translation of Golitsyn's paper, Klotz (1915) revised the seismic energy estimate slightly upwards, to 7×10^{23} erg, but offered no definitive comment on the matter of the possible trigger. By contrast, Jeffreys (1923) argued that Golitsyn's (1915) computation was erroneous, since he had not taken into account the decay of Rayleigh wave amplitudes away from the Earth's surface, which invalidates the integration over the focal hemisphere. Having corrected the calculation, and performed a cylindrical, rather than spherical, integration around the focus, Jeffreys (1923) came up with a considerably lower value of the seismic energy of the Rayleigh waves of only 1.8×10^{21} erg. To this he added a much smaller contribution from the S waves, estimated at 9×10^{19} erg.

In the context of modern theories and energy computations, we can point to a number of fatal shortcomings in Jeffreys' (1923) arguments, the most obvious one being that he assigns more energy to Rayleigh waves than to S waves. Indeed, we now know that most of the energy radiated by a seismic source is initially carried by high-frequency S waves, but those attenuate so fast in the far field that their contribution must be calculated by scaling that of the less attenuated P waves (Boatwright and Choy 1986); note that Jeffreys (1923) neglects anelastic attenuation altogether. Furthermore, he uses the model of a harmonic oscillator (despite expressing some reservations in this respect), whereas modern computations using digital data show that the major contribution to the energy integral at teleseismic distances is usually around 1 Hz. In his surface wave calculation, Jeffreys (1923) uses a single period of 14 s, and by ignoring other spectral contributions, once again underestimates the final energy of the wave, by a factor of about six with respect to the theoretical value predicted for an earthquake of that size [Okal (2003); Eq. (45), p. 2209]. In short, Golitsyn (1915) was grossly overestimating the Rayleigh energies, but we now know that they carry only a fraction of the seismic energy released, so that in the end, his result might have been correct, while Jeffreys (1923) was underestimating both Rayleigh and S energies, by considering only single frequencies. A scientific exchange between Golitsyn and Jeffreys would certainly have been enlightening, but unfortunately, Prince Golitsyn died of natural causes on 17 May (n.s.) 1916, aged only 54.

Incidentally, we now understand that the whole argument about causality was totally flawed, since the two phenomena considered (the earthquake and the rockslide) express the release of two forms of potential energy of a different nature, one elastic and the other gravitational. If one phenomenon simply *triggers* the other, the relative amounts of energy released by the two processes are unrelated, since they come from different energy reservoirs.[2] As such, an earthquake could conceivably trigger a landslide more energetic than itself (this is possibly what happens during "orphan" slides for which the triggering mechanism is simply too small to be detected), and the converse might also be envisioned, i.e., a landslide triggering a more energetic earthquake in a tectonically ripe environment.

Indeed, the interpretation of the 1911 Sarez earthquake by Golitsyn (1915) and Jeffreys (1923) as being due to the rockslide was questioned both by Oldham (1923), who argued that the earthquake source was normal (and in

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[2] These remarks illustrate scientifically the popular expression "the straw that broke the camel's back."

particular could not be superficial) on account of its isoseismal distribution and of its aftershock sequence, and by Macelwane (1926), based on a comparison with the case of the Frank, Alberta slide of 29 April 1903, for which no evidence of triggered seismicity could be found (admittedly for a significantly smaller, if still massive, rockslide, and during the dawn years of instrumental seismology). Jeffreys (1923) discusses in considerable detail the transfer of energy from a small body (the slide) falling on a big one (the Earth), and proposes a figure of 1/300 for its efficiency. His approach is correct only under his assumption "that the blow to the ground caused by the fall might have been the cause of the seismic disturbance," and thus that the elastic energy carried by the seismic waves was traceable to the deformation of the Earth upon the impact of the slide, and not to the release of tectonic strain independently accumulated during the interseismic cycle. The low efficiency of this mode of triggering was verified in the case of the collapse of the World Trade Center on 11 September 2001, for which Kim et al. (2001) proposed a value of between 10^{-4} and 10^{-3}. Incidentally, those authors also verified that the character of the seismic waves and the geometry of the spreading area differed significantly from those of a traditional earthquake, thus supporting in retrospect Oldham's (1923) criticism of Golitsyn's model for the 1911 Sarez events.

4. A few years later, in the second edition of *The Earth*, Jeffreys (1929) listed several additional values of earthquake energies, apparently all obtained from S waves: 10^{21} erg for the Montana earthquake of 28 June 1925, only 10 times smaller than suggested by (1) based on the magnitude of $6\frac{3}{4}$ later assigned by Gutenberg and Richter (1954); 5×10^{16} erg for the much smaller Hereford, England earthquake of 15 February 1926, deficient by a factor of 200 when applying (1) to the earthquake's present magnitude estimate, $M_L = 4.8$ (Musson 2007); and about 10^{19} erg for the Jersey event of 30 July 1926, for which no definitive magnitude is available. He also lists a value of 5×10^{16} erg for the Oppau, Germany explosion of

ammonium-nitrate-based fertilizer on 21 September 1921 (Wrinch and Jeffreys 1923), although we would nowadays question the concept of using S waves to quantify the source of an explosion.

5. In 1935, Richter published his landmark paper introducing the concept of magnitude, in which he cautioned that "[its] definition is in part arbitrary; an absolute scale, in which the numbers referred directly to shock energy [...] measured in physical units, would be preferable" (Richter 1935). Notwithstanding this disclaimer, Richter could not resist the temptation of relating his newly defined magnitude scale to physical units, using Jeffreys' (1929) energy estimate for the 1925 Montana earthquake. Since the latter was outside the domain of his original study (limited to California and Nevada), Richter assigned it $M = 7.5$ based on an interpretation of its isoseismals, and proceeded to scale magnitude to energy. While not expressed verbatim, the relation

$$\log_{10} E = 2M + 6 \qquad (8)$$

can be inferred from the several examples discussed on pp. 26–27 of Richter (1935). The slope $Q = 2$ in Eq. (8) is not explained, probably because it looked obvious to Richter that energy should scale as the square of amplitude, under the model of a harmonic oscillator.

6. In collaboration with Richter, Gutenberg wasted no time expanding the concept of magnitude, and within one year the two Caltech scientists had published the third in their series of "On Seismic Waves" papers (Gutenberg and Richter 1936), in which they extended the concept of magnitude to teleseismic distances and thus to earthquakes worldwide, using exclusively surface waves. In parallel, they proposed the first formal relation between magnitude and energy [their Eq. (15) p.124]:

$$\log_{10} E = 2M + \log_{10} E_0, \qquad (9)$$

E_0 being the energy of a shock of magnitude 0, "the smallest ones recorded" (of course we now know that events of negative magnitude can exist and be recorded). They also laid the foundations

for the extension of the magnitude concept to teleseismic distances, in the process revising down the magnitude of the 1925 Montana earthquake to 6.8 [later transcribed as $6\frac{3}{4}$ in Gutenberg and Richter (1954)]. This has the effect of increasing the constant 6 in (8) [or E_0 in (9)], but Gutenberg and Richter (1936) do not publish a definitive value of E_0, indicating rather that it ranges between 10^7 and 3×10^8 erg. On the other hand, they specifically justify the slope $Q = 2$ in (9), stating "Since the magnitude scale is logarithmic in the amplitudes, doubling the magnitudes gives a scale logarithmic in the energies."

Note that, up to this stage, Gutenberg and Richter do not compute their own energies based on any personal analytical approach, but rather use published values, e.g., from Reid (1910) or Jeffreys (1929); while they write "Energies have been found for earthquakes by many investigators using different methods," the modern reader is left at a loss to figure out exactly who these scientists were and what their methods might have been.

7. This situation apparently changes in the next few years, resulting in the first compilation of magnitudes derived from individual measurements of ground motion, as part of the first version of *Seismicity of the Earth*, published as a "Special Paper" of the Geological Society of America (Gutenberg and Richter 1941), although earthquakes remain grouped in "magnitude classes," from a ($M \approx 5$) to e ($M \approx 8$).

8. Gutenberg and Richter's (1942) next paper on the subject makes a number of fundamental breakthroughs. A critical discussion of the analytical parts of this paper will be given in Sect. 4.1. We present here only the general milestones in that contribution. First, the authors work out a detailed expression of the energy of the source based on a model of the vertical energy flux at the epicenter, expressed as a function of the maximum acceleration a_0 observed above a point source buried at depth h [their Eq. (27) p. 178]:

$$\log_{10} E = 14.9 + 2\log_{10} h + \log_{10} t_0 + 2\log_{10} T_0 + 2\log_{10} a_0. \tag{10}$$

where t_0 is the duration of the signal, and T_0 its (dominant) period. In this respect, and as indicated by the title of their paper, the authors are clearly motivated by relating magnitude to maximum intensities in the epicentral area, which are generally related to accelerations, rather than displacements. Similarly, the experimental data available to them in epicentral areas came primarily in the form of strong-motion accelerograms, hence the emphasis on acceleration a_0, rather than displacement A_0, in (10). Second, and perhaps more remarkably, they propose that parameters such as the duration t_0 or the prominent period T_0 of the wavetrains are not constant but rather vary with the size of the earthquake. As such, their study represents the first introduction of the concept of a *scaling law* for the parameters of a seismic source. The authors suggest that $\log_{10} t_0$ varies as $\frac{1}{4}M$ [their Eq. (28)], albeit without much justification. In modern terms, t_0 would be called a *source duration*, controlled by the propagation of rupture along the fault, and hence expected to grow like $M_0^{1/3}$, which would be reconciled with Gutenberg and Richter's (1942) Eq. (28) if $\log_{10} M_0$ were to grow like $\frac{3}{4}M$, an unlikely behavior. The fundamental empirical observation in Gutenberg and Richter (1942) is that the maximum acceleration a_0 at the epicenter grows slower with magnitude than the amplitude of ground motion used to compute M [their Eq. (20) p. 176]:

$$M = 2.2 + 1.8\log_{10} a_0, \tag{11}$$

from which they infer a slope of 0.22 between $\log_{10} T_0$ and M [their Eq. (32), p. 179]. T_0 represents the dominant period of an accelerogram at the epicenter, which in modern terms may be related to the inverse of a *corner frequency*, itself controlled by a combination of source duration and rise time, and thus T_0 would be expected to grow like $M_0^{1/3}$ (Geller 1976), which is reconciled with Eq. (32) of Gutenberg and Richter (1942) if $\log_{10} M_0$ were to grow like $\frac{2}{3}M$, again an unlikely behavior. When substituting the dependence of t_0 and T_0 with M into their

Eq. (27), the authors finally obtain their Eq. (34) p. 179:

$$\log_{10} E = 8.8 + 2 \log_{10} h + 1.8M \qquad (12)$$

as a replacement for the previous version featuring a slope $Q = 2$ (Gutenberg and Richter 1936). Note that this formula would correspond to (10) for $h = 40$ km, which is significantly greater than the typical depth of the Southern California earthquakes that constituted the dataset used by the authors for most of their investigations, and would even be greater than the probable depth of focus of most of their teleseismic events. They correctly assign the reduction in slope to issues nowadays described as evolving from *source finiteness*, even though these effects remain underestimated by modern standards.

Gutenberg and Richter (1942) then proceed to estimate (only to the nearest order of magnitude) energies for 17 large earthquakes, based on intensity reports at their epicenter, or on their radius of perceptibility, both approaches being related to epicentral ground acceleration, and the latter reminiscent of Milne's (1898) methodology. The values proposed are significantly overestimated (reaching 10^{26} to 10^{27} erg), which may result from underestimating the effect of source finiteness, ignoring in particular the existence of several corner frequencies, a phenomenon leading to a saturation of acceleration [ground accelerations in excess of one g are not observed systematically for great earthquakes, but rather in certain tectonic environments in the case of even moderate, but "snappy," shocks (e.g., Fry et al. 2011)]. As a result, the growth of E with M [the slope $Q = 1.8$ in Eq. (12)] remains too strong, and this eventually overpredicts energy values, especially for large events. However, from a *relative* standpoint, it is remarkable that, in addition to the truly great earthquakes in Assam (1897) and San Francisco (1906), the three events given the strongest energies by Gutenberg and Richter (1942) are the shocks of 1926 off Rhodos, 1939 in Chillán, Chile, and 1940 in Vrancea, Romania. The

Chillán event has been shown by Okal and Kirby (2002) to feature an anomalously high energy-to-moment ratio, a property expected to be shared by the other two on account of their location as intermediate-depth intraslab events (Radulian and Popa 1996; Ambraseys and Adams 1998). Finally, and in retrospect, a significant problem with the multiple regression (10) underlying Gutenberg and Richter's (1942) model is that it predicts a logarithmic discontinuity in E as the source reaches the surface ($h \to 0$). This limitation reflects the simplified model of a point source, which is rendered invalid as soon as the fault's width W and length L become comparable to h.

9. In 1945, Gutenberg published three papers establishing the computation of magnitudes from surface and body waves on a stronger operational basis. First, in Gutenberg (1945a), he formalized the calculation of a surface-wave magnitude M_s by introducing the distance correction $1.656 \log_{10} \Delta$ [his Eq. (4)]. This slope, still of an empirical nature, is significantly less than suggested by Gutenberg and Richter (1936); for example, the 42 points at distances less than $150°$ on their Fig. 6 (p. 120) regress with a slope of $(-2.08 \pm 0.09) \log_{10} \Delta$ (or (-1.94 ± 0.15) for $\Delta < 55°$). The new slope (rounded to 1.66) was to be later inducted (albeit after considerable debate) into the Prague formula for M_s (Vaněk et al. 1962); while never derived theoretically, it was justified as an acceptable empirical fit to a modeled decay of 20-s Rayleigh wave amplitudes with distance (Okal 1989).

In the second paper, Gutenberg (1945b) used the concept of geometrical spreading, initially described by Zöppritz [and written up as Zöppritz et al. (1912) following his untimely death], to extend to teleseismic distances the calculation of an energy flux pioneered at the epicenter by Gutenberg and Richter (1942), thus defining a magnitude from the body-wave phases P, PP and S. The most significant aspects of this paper are (1) the difficulty of the author to obtain both local and distant values of magnitudes for the same event; (2) the necessity to invoke station corrections reflecting site responses; (3)

the relatively large values of the periods involved (up to 7 s even when considering only P phases); and (4) the introduction of correction terms for magnitudes larger than seven, clearly related to the effects of source finiteness.

In the third paper, Gutenberg (1945c) attacked the problem of deep earthquakes, and produced (for P, PP, and S) the first versions of the familiar charts for the distance–depth correction, generally referred to as $Q(\Delta, h)$, but as $A(\Delta, h)$ in early papers,[3] to be applied to the logarithm of ground motion amplitude. After this correction was significantly adjusted by Gutenberg and Richter (1956b) [their Fig. 5, as compared with Gutenberg's (1945c) Fig. 2], it was to be retained in the Prague formula for m_b (Vaněk et al. 1962), and has remained to this day the standard for the computation of magnitudes from P waves. By imposing that the same earthquake should have the same magnitude when measured at different distances, it was possible, at least in principle, to obtain empirically the variation of $Q(\Delta, h)$ with distance, and in particular to lock the body-wave magnitude scale with Richter's (1935) original one. However, the dependence with depth obviously required a different approach. In the absence of a physical representation of the source by a system of forces, Gutenberg (1945c) elected to impose that shallow and deep earthquakes of similar magnitude should have the same radiated energy, which allowed him to obtain the first version of the $A(\Delta, h)$ chart (his Fig. 2), through a generalization of the concept of geometrical spreading. We now understand that this approach tacitly assumes a constant stress drop $\Delta\sigma$, which may not be realistic for deep sources at the bottom of subduction zones.

In hindsight, Gutenberg's (1945c) approach suffered from ignoring the presence of the low-velocity zone in the asthenosphere, as well as of the seismic discontinuities in the transition zone,

which result in significant distortion of slowness as a function of distance, and hence of its derivative controlling geometrical spreading. In addition, the attenuation structure of the mantle was at the time little if at all understood, which later prompted Veith and Clawson (1972) to propose a new, more streamlined, distance–depth correction, $P(\Delta, h)$, based on a more modern representation of attenuation as a function of depth; however, they elected to use a structure derived from Herrin's (1968) tables, which does not include mantle discontinuities, even though the latter were documented beyond doubt by the end of the 1960s (e.g., Julian and Anderson 1968). Note finally that Gutenberg (1945b, Eq. 18, p. 66; 1945c, Eq. (1), p. 118) introduces the ratio A/T rather than A in the computation of m_B, presumably motivated by the quest for a closer relation between magnitude and energy, the latter involving ground velocity rather than displacement in its kinetic form. This change from A to A/T is potentially very significant, since T is expected to vary with earthquake size.

10. The next decade saw the compilation of the definitive version of *Seismicity of the Earth*, published as a monograph in two subsequent editions (Gutenberg and Richter 1949, 1954), in which earthquakes are assigned individual magnitudes. In their introduction to the final edition (p. 10), the authors argued that the value $Q = 1.8$, derived in Gutenberg and Richter (1942) and used in the first one, overestimated energies, and suggested the lower value $Q = 1.6$; however, they did not revise their discussion of energy, perhaps because they felt that their new formula (5), which they had not yet formally published (and eventually would never publish), was itself not definitive.

11. In the meantime, the scientific value of the concept of magnitude as a quantification of earthquake sources had become obvious to the seismological community, and many investigators developed personal, occasionally competing, algorithms, and more generally offered comments on Gutenberg and Richter's ongoing work. Among them, Bullen (1953) suggested that

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[3] The correction is unrelated to the slope Q between magnitude and energy, and to the amplitude of ground motion A used to measure magnitudes. It also bears no relation to the quality factor Q later defined as the inverse of anelastic attenuation.

energies derived from (12) were too large for the largest magnitudes, which would go in the direction of the reduction of Q. On the other hand, following Jeffreys' (1923) criticism of Golitsyn's (1915) calculation of the energy of Rayleigh waves, Båth (1955) proposed to correct it by restricting the vertical cross-section of the teleseismic energy flux (which he evaluates at the Earth's surface) to a finite depth, H, which he took as 1.1 times the wavelength Λ. While this approach is indeed more sound than Golitsyn's, it is still limited to Rayleigh waves, and thus does not quantify the contribution of body waves, even though Båth (1955) does state that the latter is probably important. Finally, his empirically derived slope Q reverts to the value 2, presumably because of his exclusive use of narrowband instruments, essentially converting the Earth's motion into that of a harmonic oscillator. Similarly, Di Filippo and Marcelli (1950) obtained $Q = 2.14$ from a dataset of Italian earthquakes using Gutenberg and Richter's (1942) methodology, this higher value reflecting the departure of the dominant period T from the scaling proposed by Gutenberg and Richter. Sagisaka (1954) attempted to reconcile Reid's (1910) approach, based on integrating the energy of the elastic deformation, with Gutenberg and Richter's (1942) values derived from magnitudes in the case of several Japanese earthquakes (both shallow and deep), and noticed that the latter were consistently excessive, e.g., by a factor of at least 100 in the case of the great Kanto earthquake of 1923. We have proposed (Okal 1992) a seismic moment of 3×10^{28} dyn cm for that event, which under modern scaling laws, would suggest an energy of about 4×10^{23} erg, in general agreement with Sagisaka's (1954) figure, and confirming that (12) overestimates energies, at least for large events.

12. The year 1956 sees no fewer than three new publications on the subject by Gutenberg and Richter. Gutenberg (1956) and Gutenberg and Richter (1956b) constitute in particular their last efforts at trying to reconcile the various magnitude scales they had arduously built over the previous 20 years, into a single "unified"

magnitude. In this framework, they proceed to develop empirical relations between magnitude scales (and hence with energy) which become more complex, and in particular involve nonlinear terms. We now understand that, because they were measuring different parts of the seismograms at different periods and the scaling laws underlying the concept of a unified magnitude were distorted differently [different *corner frequencies* apply to different wavetrains (Geller 1976)], it was impossible for them to find a simple relation between magnitudes that would apply for all earthquake sizes. For example, we note that the relation proposed in Eq. (20), p. 134 of Gutenberg and Richter (1956a):

$$\log_{10} E = 9.4 + 2.14M - 0.054M^2 \qquad (13)$$

would regress with a slope $Q = 1.6$ between magnitudes 1 and 9, but only $Q = 1.4$ between the more usual values of 5.5 and 8.5. We note that Gutenberg (1956) elects to align a "unified" magnitude m on his body-wave m_B. This may sound surprising since we now understand that, being higher frequency than a surface-wave magnitude, m_B is bound to suffer stronger distortion from the effect of source finiteness, and saturate earlier (Geller 1976). However, as suggested in Gutenberg and Richter (1956a), Gutenberg may have been motivated by the goal of matching Richter's (1935) initial scale, which would be regarded today as a local magnitude M_L, measured at even higher frequencies, and therefore more closely related to a body-wave scale. Also, Richter's initial magnitude used torsion records with deficient response at longer periods.

In many respects, the third paper (Gutenberg and Richter 1956a) [whose preparation apparently predated Gutenberg and Richter (1956b)] stands in a class by itself, and constitutes a superb swan song of the authors' collaboration on this subject. In particular, it presents a review of previous efforts at extracting energy from seismograms, and revises the power laws relating phase duration t_0 and dominant period T_0 to magnitude. The authors now propose $\log_{10} t_0 = 0.32M - 1.4$

[rather than $0.25M - 0.7$ in Gutenberg and Richter (1942)], which would be in remarkable agreement with modern scaling laws predicting a slope of 1/3, but only in the absence of source finiteness. However, the dominant period T_0 in the seismogram is described as being essentially independent of magnitude. The authors suggest an algorithm for the computation of seismic energy from P waves ("integrating $(A_0/T_0)^2$ over the duration of large motion of the seismogram") which constitutes a blueprint for later and definitive algorithms, such as Boatwright and Choy's (1986), but of course, in the absence of digital data, they can only propose an approximate methodology (integrating the maximum value of velocity squared over an estimated duration of the phase), which by modern standards, keeps too much emphasis on representing the source as a monochromatic oscillator.

In the course of what was to be their last contribution on this topic, Gutenberg and Richter (1956a) make a number of visionary statements, notably concerning the limitations inherent in the model of a point source, which they argue make it uncertain that "there is a one-to-one correspondence between magnitude [...] and the total energy radiation." They conclude by emphasizing the necessity of a complete revision of the magnitude scale, which they claim to be in preparation, based on ground velocity (A/T) rather than amplitude, this statement reflecting once again both the pursuit of Richter's (1935) original goal of relating magnitudes to the physical units of energy, but also the common and erroneous model of a monochromatic oscillator.

Beno Gutenberg died suddenly on 25 January 1960, and Charles Richter, the inventor of the magnitude concept, never published any further contribution on the topic of magnitudes or seismic energy.

13. Developments regarding magnitude scales and the computation of seismic energy following Gutenberg's death are better known and will be only briefly summarized here. The torch was then passed to Eastern Bloc scientists. Following the International Union of Geodesy and Geophysics

(IUGG) meeting in Helsinki in 1960, Czech and Russian scientists met in Prague in 1961, and standardized the calculation of body- and surface-wave magnitudes, respectively, m_b (measured in principle at 1 s) and M_s (at 20 s) (Vaněk et al. 1962).[4] Both algorithms were based on the use of A/T, which the authors specifically justified as linked to energy, thus once again tacitly assuming the model of a monochromatic oscillator. In operational terms, a major difference arises between m_b and M_s: because of the prominence of 20-s waves strongly dispersed along oceanic paths, and suffering little attenuation in the crust, and of the universal availability of narrowband long-period instruments peaked around its reference period, M_s has indeed been measured close to 20 s, while measurements of m_b have often been taken at periods of several seconds, which considerably affects not only its relation to physical source size (i.e., seismic moment and hence energy), but also its saturation for large earthquakes. As a consequence, the Prague-standardized M_s has been more comparable to previous versions of surface-wave magnitudes than its m_b counterpart may have been to previous body-wave scales such as Gutenberg's (1945b) m_B, for which measurements were often taken at more variable periods, apparently as long as 7 s.

14. In the meantime, an interesting development had taken place in the then Soviet Union with the establishment of the so-called K-class scale for regional earthquakes. We refer to Rautian et al. (2007) and Bormann et al. (2012) for detailed reviews, and will only discuss here a number of significant points. That work originated with the need to compile and classify the very abundant seismicity occurring in the area of Garm, Tajikistan, notably in the aftermath of the Khait earthquake of 10 July 1949. It is remarkable that, even though it was conducted under what amounted to material and academic autarky

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[4] This landmark paper was published simultaneously in Moscow (Vaněk et al. 1962) and Prague (Kárník et al. 1962). The different listing of coauthors reflects the use of different alphabetical orders in the Cyrillic and Latin alphabets.

(Hamburger et al. 2007), it nevertheless proceeded with great vision, and was rooted, at least initially, in the principles which would later constitute the foundations of modern algorithms using digital data (e.g., Boatwright and Choy 1986).

Initial work on the K-class was clearly intent on deriving a physically rigorous measurement of energy from seismic recordings by setting an a priori relation with energy (Bune 1955), later formalized as

$$K = \log_{10} E \qquad (14)$$

with E in joules (Rautian 1960). This approach differed fundamentally from Richter's (1935) empirical definition of magnitude, and indeed pursued the work of Golitsyn (1915) by seeking to compute an energy flux at the receiver (Bune 1956). It is interesting to note that Bune clearly mentions the need to work in the Fourier domain, but in the absence of digital data (and of computational infrastructure), he resorted to a time-domain integration over a succession of individual wavetrain oscillations, which may be justifiable under the combination of Parseval's theorem and the presence of strong systematic dispersion during propagation; however, the latter may not be sufficiently developed at the short distances involved. Faced with these challenges, Rautian (1960) reverted to the simplified practice of adding the absolute maximum amplitudes of P and S traces, in order to define an adequate "amplitude" to be used in the computation of the energy flux. She also pointedly recognized the influence of instrumentation, and later the variability required when exporting the algorithm to geologically different provinces (e.g., Fedotov 1963; Solov'ev and Solov'eva 1967), both of which can be attributed to a filtering effect (by instrument response and regional anelastic attenuation), before interpreting a complex source spectrum through a single number in the time domain.

Attempts to relate K-class values to magnitudes (as measured in the Soviet Union) were marred by the fact that the former was built for small events recorded at regional distances, while the latter, based on the application of the Prague formula to Love waves (Rautian et al. 2007), tacitly assumed larger shocks recorded at teleseismic distances. Nevertheless, the initial work of Rautian (1960) and systematic formal regressions later performed by Rautian et al. (2007) indicate an average slope $Q = 1.8 \pm 0.3$ [note, however, that in an application to larger earthquakes, Rautian (1960) suggested the use of a "total" seismic energy E_0, growing slower than E (in fact like $E^{2/3}$; her Eq. (19)), leading to a lower value of $Q = 1.1$]. The origin of the relatively high value $Q = 1.8$ is unclear, but probably stems from the simplified algorithm used by Bune (1956) and Rautian (1960), which may amount to narrow-bandpass filtering, thus approaching the conditions of a monochromatic signal ($Q = 2$).

In retrospect, the clearly missing element in the K-class algorithm is the key link between the energy flux at the station and the radiation at the source, which is now understood in the model of ray tubes, and quantified using a formula expressing geometrical spreading, e.g., in the algorithm later developed by Boatwright and Choy (1986). This concept, initially published as Zöppritz et al. (1912), could not be applied to the regional phases (P_n, S_n, L_g) on which the K-class was built, which motivated Rautian (1960) to develop other, more empirical algorithms.

15. Further significant progress could be made in the early 1960s on account of several theoretical developments. First, the double-couple was introduced by Vvedenskaya (1956), as the physical representation of a dislocation along a fault in an elastic medium, and later formalized by Knopoff and Gilbert (1959); its amplitude, the scalar moment M_0, materializes the agent, measurable in physical units, to which Richter (1935) had lamented he had no access. In addition, Ben-Menahem (1961) published a landmark investigation of the effect of source finiteness on the spectrum of seismic surface waves, introducing the concept of directivity, which was to prove critical in resolving the "energy paradox" exposed in Sect. 2.1. Based on the representation

theorem, and considering a source of finite dimension, Haskell (1964) gave the first expression for the energy radiated into P and S waves by a finite source of arbitrary geometry, which he found proportional to

$$E \propto \frac{W^2(\Delta u)^2 L}{\tau^2}, \qquad (15)$$

where we have rewritten his Eq. (39) (p. 1821) with the more usual notation W for fault width, L for fault length, Δu for fault slip, and τ for rise time. Under generally accepted scaling laws (Aki 1967), we anticipate that E would indeed be proportional to M_0, in agreement with Vassiliou and Kanamori's (1982) later work. Haskell's paper is remarkable in that it is the first one to express seismic energy in the frequency domain by means of a Fourier decomposition. By then, efficient Fourier-transform algorithms were becoming available (Cooley and Tukey 1965), but digitizing analog records at time samplings adequate to compute energy fluxes remained a formidable challenge, which would be resolved only with the implementation of analog-to-digital converters in the 1970s (Hutt et al. 2002); it is not fortuitous that the only spectral data published in Haskell (1964) and its statistical sequel (Haskell 1966) are limited to $f < 12$ mHz [his Fig. 2, reproduced from Ben-Menahem and Toksöz (1963), and incidentally relating only to source phase]. The same improvement in computational capabilities in the 1960s had led to the systematic development of ray-tracing techniques which revitalized the estimate of energy fluxes in the far field [whose idea, we recall, can be traced all the way back to Mendenhall (1888)], based in particular on the concept of geometrical spreading defined by Zöppritz et al. (1912). In particular, Wu (1966) laid the bases for the computation of seismic energy from digitized teleseismic body waves, but in practice, it could be applied only to long-period records until digital data became available in the 1970s. The state of affairs in the mid-1960s is detailed in Båth's (1967) review paper, which rather surprisingly does not mention Ben-Menahem's

(1961) work on source finiteness, which was to play a crucial role in any understanding of magnitude and energy for large earthquakes.

16. By contrast, Brune and King (1967) addressed the question of the influence of finiteness for large earthquakes when using conventional magnitudes measured at insufficient periods (e.g., M_s), and embarked on computing "mantle" magnitudes using 100-s Rayleigh waves (their proposed M_M, which we call here M_{100}), thus becoming the first to propose a systematic use of mantle waves to quantify large earthquake sources. They documented a dependence of M_{100} on M_s (actually more precisely on an undefined M) in the shape of a stylized "S", whose intermediate regime corresponds to events with initial corner periods between 20 and 100 s. Brune and Engen (1969) complemented this study with measurements on Love waves, this time taken in the frequency domain in order to separate individual periods in the absence of dispersion. Remarkably, Brune and Engen (1969) recognized two striking outliers in their $(M_{100} : M)$ dataset: they found the 1933 Sanriku event deficient in M_{100} despite a record $M = 8.9$ assigned by Richter (1958); we have documented this shock as a "snappy" intraplate earthquake, which violated the scaling laws tacitly implied by Richter when converting a long-period magnitude into a "unified" one based on a short-period algorithm [see Okal et al. (2016) for a detailed discussion]. On the opposite side, Brune and Engen (1969) noticed that the 1946 Aleutian earthquake, now known as a "tsunami earthquake" of exceptional source slowness (Kanamori 1972; López and Okal 2006), featured a mantle magnitude significantly larger than its standard M_s. In a visionary statement, which unfortunately remained largely unnoticed at the time, they stressed the potential value of a mantle magnitude in the field of tsunami warning (Brune and Engen 1969; p. 933).

17. The connection between directivity, as introduced by Ben-Menahem (1961), and full saturation of magnitude scales measured at constant periods was described in the now classic papers by Kanamori and Anderson (1975) and

Geller (1976). As mentioned in Sect. 2, Geller's last set of (unnumbered) equations on p. 1520 detail the relationship between M_s, measured at 20 s, and $\log_{10} M_0$, and in particular the evolution of the slope between the latter and former from a value of 1 at low magnitudes, through 1.5 over a significant range of "large" earthquakes ($6.76 \leq M_s \leq 8.12$), a narrow interval where a slope of 3 is predicted, and a final saturation of M_s at a value of 8.22. We reproduce these relations, complete with moment ranges, here:

$$M_s = \log_{10} M_0 - 18.89 \quad \text{for}$$
$$\log_{10} M_0 \leq 25.65 \, (M_s \leq 6.76), \tag{16a}$$

$$M_s = \frac{2}{3}(\log_{10} M_0 - 15.51) \quad \text{for}$$
$$25.65 \leq \log_{10} M_0 \leq 27.69 \, (6.76 \leq M_s \leq 8.12), \tag{16b}$$

$$M_s = \frac{1}{3}(\log_{10} M_0 - 3.33) \quad \text{for} \quad 27.69 \leq \log_{10} M_0$$
$$\leq 28.00 \, (8.12 \leq M_s \leq 8.22), \tag{16c}$$

$$M_s = 8.22 \quad \text{for} \quad \log_{10} M_0 \geq 28.00. \tag{16d}$$

The combination of these equations with the relationships derived between m_b and M_s (Geller 1976; first set of unnumbered equations, p. 1520) leads to the following four-segment expression for the variation of m_b with moment:

$$m_b = \log_{10} M_0 - 17.56 \quad \text{for}$$
$$\log_{10} M_0 \leq 21.75 \, (m_b \leq 4.19), \tag{17a}$$

$$m_b = \frac{2}{3}(\log_{10} M_0 - 15.47) \quad \text{for}$$
$$21.75 \leq \log_{10} M_0 \leq 23.79 \, (4.19 \leq m_b \leq 5.55), \tag{17b}$$

$$m_b = \frac{1}{3}(\log_{10} M_0 - 7.16) \quad \text{for} \quad 23.79 \leq \log_{10} M_0$$
$$\leq 25.16 \, (5.55 \leq m_b \leq 6.00), \tag{17c}$$

$$m_b = 6.00 \quad \text{for} \quad \log_{10} M_0 \geq 25.16. \tag{17d}$$

These relations were obtained assuming a constant stress drop, a constant aspect ratio (L/W) of the fault, and constant particle and rupture velocities. Equations (16) and (17) are plotted in Fig. 1.

18. In his landmark paper, Kanamori (1977) introduced the concept of "moment magnitude" M_w by casting an independently obtained bona fide scientific measurement of the seismic moment M_0 (in dyn cm) into a magnitude scale through

$$M_w = \frac{2}{3}\left[\log_{10} M_0 - 16.1\right]. \tag{18}$$

This definition of M_w seeks to (1) relate M_w to earthquake energy (hence the subscript "w"); and (2) make its values comparable to those (M) previously published, notably by Gutenberg and Richter. In doing so, it specifically assumes a ratio of 0.5×10^{-4} between energy and moment [Kanamori 1977, Eq. (4′) p. 2983], and also "Gutenberg and Richter's energy–magnitude relation" (1). While the former can be derived under scaling laws, we have seen that the latter lacked a satisfactory theoretical derivation. By further seeking to ensure the largest possible continuity between M_w and traditional M, Kanamori (1977) forces the factor 2/3 into (18) and thus tacitly assumes that the dataset of M is always taken in the size range where the magnitude has started to feel the effects of source finiteness, characterized by a slope of 1.5 in Fig. 1. This was further developed in the case of local magnitudes M_L by Hanks and Kanamori (1979) and can also be applied conceptually to the case of the body-wave magnitude m_b. The relationship (18) is then predicted to give an estimate M_w approaching a classical magnitude as long as the latter is computed exclusively in a domain where it has started to be affected by source finiteness, but with only *one* corner frequency (that relating to fault length L) lower than the reference frequency of that magnitude scale, thus resulting in a slope of 1.5 in Fig. 1. In other words, M_w should coincide with a traditional magnitude measurement if and only if that magnitude is M_s for reasonably large earthquakes ($25.65 \leq \log_{10} M_0 \leq 27.69$), m_b for smaller events ($21.75 \leq \log_{10} M_0 \leq 23.79$), and presumably M_L at even smaller moments.

Otherwise, and notably for smaller events (e.g., if using M_s for magnitude 5 events), one should expect a discrepancy between M_w and M_s; this is indeed the subject of the "bias" reported by Ekström and Dziewoński (1988). In general terms, Fig. 1, on which we have superimposed (in solid green) the relationship predicted by (18), features such a trend, but the values of M_w are smaller, by about 0.35 logarithmic units, than predicted by the dotted red and dashed blue lines. Similarly, in the regime for smaller-size events (slope of 1), the constant (18.89) relating M_s to $\log_{10} M_0$ in (16a) is smaller than derived theoretically by Okal (1989) (19.46, a difference of 0.57 units) or obtained experimentally by Ekström and Dziewoński (1988) (19.24; 0.35 units). This observation is traceable to the modeling of the $M_s : M_0$ relationship by Geller (1976), copied here as Eq. (16); note in particular that the fit of these relations to the dataset in his Fig. 7 deteriorates significantly for earthquakes in the moment range 10^{27} to 10^{28} dyn cm. A possible explanation is Geller's use of a relatively high $\Delta\sigma = 50$ bar as an average stress drop. For this reason, we prefer to replace Eq. (16) with

$$M_s = \log_{10} M_0 - 19.46 \quad \text{for} \\ \log_{10} M_0 \leq 26.22 \, (M_s \leq 6.76), \quad (19a)$$

$$M_s = \frac{2}{3}(\log_{10} M_0 - 16.08) \quad \text{for} \\ 26.22 \leq \log_{10} M_0 \leq 28.26 \, (6.76 \leq M_s \leq 8.12), \\ (19b)$$

$$M_s = \frac{1}{3}(\log_{10} M_0 - 3.90) \quad \text{for} \\ 28.26 \leq \log_{10} M_0 \leq 28.56 \, (8.12 \leq M_s \leq 8.22), \\ (19c)$$

$$M_s = 8.22 \quad \text{for} \quad \log_{10} M_0 \geq 28.56, \quad (19d)$$

shown in Fig. 2 as the thick red line superimposed on the background of Geller's (1976) Fig. 7. Note that a better fit is provided to interplate earthquakes (solid dots), especially in the range of moments 10^{27}–10^{28} dyn cm, characterized by the slope of 2/3 in (19b); upon
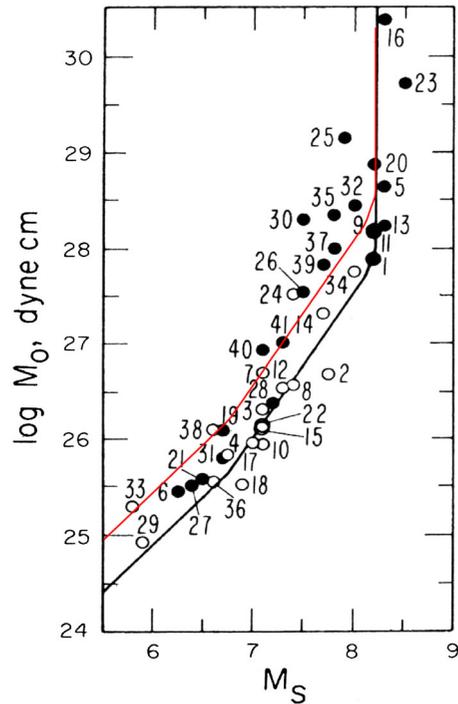

Figure 2
Reproduction of Fig. 7 of Geller (1976), with our preferred relationship (19) between M_s and $\log_{10} M_0$ superimposed in red

reduction of stress drop, corner frequencies are in principle lowered, which results in a slight displacement of saturation effects to higher moments, and a better agreement with (18). We similarly replace (17) with

$$m_b = \log_{10} M_0 - 18.18 \quad \text{for} \\ \log_{10} M_0 \leq 22.36 \, (m_b \leq 4.19), \quad (20a)$$

$$m_b = \frac{2}{3}(\log_{10} M_0 - 16.08) \quad \text{for} \\ 22.36 \leq \log_{10} M_0 \leq 24.41 \, (4.19 \leq m_b \leq 5.55), \\ (20b)$$

$$m_b = \frac{1}{3}(\log_{10} M_0 - 7.76) \quad \text{for} \\ 24.41 \leq \log_{10} M_0 \leq 25.76 \, (5.55 \leq m_b \leq 6.00), \\ (20c)$$

$$m_b = 6.00 \quad \text{for} \quad \log_{10} M_0 \geq 25.76. \quad (20d)$$

In Fig. 3, it is clear that the fit to M_s and m_b in the ranges where they feature a slope of 2/3 is much improved; we also achieve a small
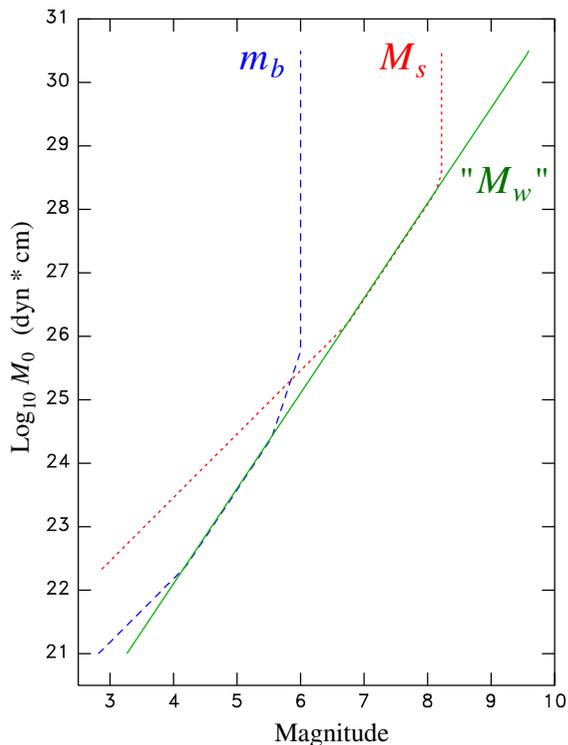
Figure 3

Same as Fig. 1, with our preferred relationships (19) and (20) for m_b and M_s. Note the better agreement of M_w with both, in the ranges where either has started to saturate

reduction in the gap between moments separating regimes for which M_w coincides with either M_s or m_b.

In this context, and in retrospect, it is unfortunate that Brune and Engen (1969) would not (or could not) push their measurements to periods longer than 100 s. This should have been possible at least for the largest earthquakes they studied, and would probably have led them to the explanation of the full saturation of magnitude scales, given that even M_{100} will eventually be affected by finite source dimensions (bringing the second elbow in the stylized "S" in their Fig. 3), and then fully saturate for even larger events. In this respect, note that the dataset plotted on that figure uses as abscissæ "Magnitudes" which cannot be strict M_s values, the latter saturating around 8.2 (Geller 1976, Fig. 7, p. 1517), and that the stylized "S" fails to take into account the effect of subsequent corner frequencies, for both M_s and M_{100}, which is expected to lead to the

saturation of both. This is shown in Fig. 4, where we plot a theoretical version of their relationship, obtained by adapting Eq. (19) to a reference period 5 times longer (which simply amounts to multiplying all elbow moments by a factor of 125). However, the fully saturated value $M_{100} = 9.77$ would occur at $M_0 = 1.2 \times 10^{30}$ dyn cm, which would make it essentially unobservable.

19. Vassiliou and Kanamori (1982) applied Haskell's (1964) concept to derive energy estimates by extracting the seismic moment M_0 and the time-integrated value of the source rate time function squared (I_t in their notation) from hand-digitized analog long-period records of teleseismic body waves. They concluded that E/M_0 was essentially constant for shallow (and even a few deep and intermediate) sources, but suggested a slope $Q = 1.8$ when regressing their estimates of $\log_{10} E$ against published values of M_s. This is probably due to the regression sampling into a range of moments ($\geq 10^{27.9}$) where severe saturation affects M_s and drives it away from its domain of variation as $\frac{2}{3} \log_{10} M_0$ (their Fig. 9a and Table 1). It is noteworthy that Vassiliou and Kanamori (1982) were the first to publish logarithmic plots of energy-to-moment datasets, later used systematically by Choy and Boatwright (1995) and Newman and Okal (1998). Without access to high-frequency digital data, they elected to directly estimate the ratio E/M_0^2 [their Eq. (3) p. 373] from what amounts to modeling the shape of long-period body waves, which were at the time the only ones they could hand-digitize from paper records. However robust the procedure may be with respect to parameters in the simple tapered-boxcar source model they use (their Fig. 1), their approach will perform poorly with complex, jagged sources such as those characteristic of a number of tsunami earthquakes (Tanioka et al. 1997; Polet and Kanamori 2000).

20. Following the deployment of short-period and broadband digital networks in the 1970s, the landmark paper by Boatwright and Choy (1986) finally regrouped all necessary ingredients for the routine computation of radiated energy, including the far-field energy flux approach (Wu 1966),
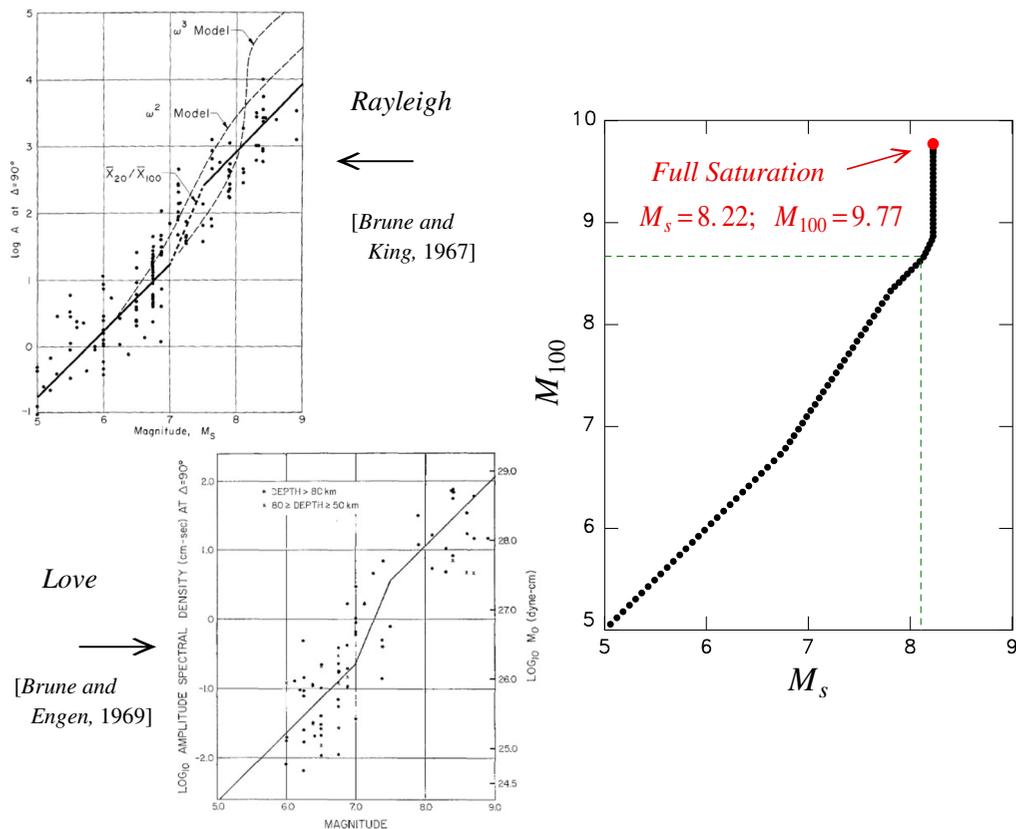
Figure 4

Left: Variation of mantle magnitudes versus conventional ones, reproduced from Brune and King (1967) (Rayleigh; *top*) and Brune and Engen (1969) (Love; *bottom*). Note that the abscissæ are probably not strict M_s values, which are supposed to saturate around 8.2. *Right:* Theoretical behavior of a 100-s magnitude versus 20-s M_s as predicted using Geller's (1976) model. The dashed line reproduces the domain of study of Brune's papers, in agreement with the stylized "S" curves at left. Note the eventual saturation of M_{100} at a value of 9.77 ($M_0 = 1.2 \times 10^{30}$ dyn cm)

a computation in the Fourier domain as suggested by Haskell (1964), and adequate corrections for geometrical spreading, anelastic attenuation, and focal mechanism orientation. It set the stage for the systematic computation of radiated energies, the catalog of Choy and Boatwright (1995) providing the first extensive demonstration of a generally constant ratio E/M_0. Outliers to this trend, few in number but of critical importance in terms of seismic or tsunami risk and providing insight into ancillary problems in plate tectonics, were later identified by Choy et al. (2006). Newman and Okal (1998) developed a simplified alternative to Boatwright and Choy's (1986) algorithm, allowing rapid computation of $\Theta = \log_{10}(E/M_0)$, a robust

estimate of the slowness of a seismic source, which was implemented as part of tsunami warning procedures (Weinstein and Okal 2005). Finally, Convers and Newman (2013) have combined the measurement of radiated energy with that of rupture duration, both obtained from P waves, to identify as rapidly as possible anomalously slow events bearing enhanced tsunami risk; a similar approach can be found in the definition of Okal's (2013) parameter Φ.

21. Later developments in the study of the energy radiated by seismic sources largely transcend the mainly historical scope of the present paper, and we will only review them succinctly.

Using a wide variety of sources from the largest megaearthquakes to microearthquakes induced

during shaft excavation in granite (Gibowicz et al. 1991), McGarr (1999) and Ide and Beroza (2001) documented that, remarkably, the general constancy of E/M_0 can be extended over 17 orders of magnitude of seismic moment. However, as summarized, e.g., by Walter et al. (2006), later studies failed to bring a firm consensus for either a constant stress drop across the range of significant earthquake sources, as suggested, e.g., by Prieto et al. (2004) and more recently Ye et al. (2016), or for a detectable increase of "scaled energy" E/M_0 for large earthquakes (e.g., Mayeda et al. 2005). The origin of this disparity of results remains obscure, but may be rooted in the difficulty (or impossibility) to obtain adequate, universal models of attenuation at high frequencies (Sonley and Abercrombie 2006), with Venkataraman and Kanamori (2004) also mentioning the possible effect of source directivity in biasing the computation of radiated energy for large earthquakes.

4. A Discussion of Gutenberg and Richter's Derivations

The timeline in Sect. 3 provides a comprehensive examination of the history of estimates of the energy radiated by seismic sources, as well as of the developments of the concept of magnitudes. We return here to the origin of Eq. (1) and specifically to the derivation of the slope $Q = 1.5$ implicit in Gutenberg and Richter (1956a, b), and of the earlier value $Q = 1.8$ (Gutenberg and Richter 1942). The process by which these values were obtained results from a combination of parameters (σ_i and μ_j, see below) generally expressing power laws controlling the relative growth with earthquake size of various physical quantities; these parameters were usually obtained in an empirical fashion by Gutenberg and Richter, but can now be explored in the context of seismic source scaling laws. Tables 1 and 2 summarize their values and properties, in particular their poor robustness.

As explained in Sect. 2.2, radiated energy is proportional to seismic moment M_0, and this was verified eventually from datasets such as Choy and

Boatwright's (1995) or Newman and Okal's (1998). On the other hand, the concept of magnitude, which measures the logarithm of ground displacement, should be proportional to $\log_{10} M_0$, at least in the absence of saturation effects due to source finiteness. The combination of these two remarks should lead to a theoretical value of $Q = 1$.

We start by noting that energy is carried mostly by high-frequency body waves, and as such should be measured on high-frequency seismograms. On a global scale, this restricted early authors to measurements of acceleration by strong-motion instruments, at least initially (in the 1930s and 1940s). It is probable that, partly because of this instrumental restriction and partly because their model to trace back the energy flux to a point source was developed only above the hypocenter, all their energy calculations were performed at the epicenter (with the subscript zero on all relevant variables, such as A_0, a_0, etc.). This remains surprising since short-period instruments providing high performance in the far field had been developed at Caltech (Benioff 1932), and were operating routinely by the late 1930s. Their records were indeed used by Gutenberg (1945b, c) to develop his body-wave magnitude m_B. One can only speculate as to why Gutenberg and Richter did not embark on a teleseismic measurement of radiated energy, especially since Gutenberg was obviously cognizant of the concept of geometrical spreading, having written up (with Geiger) the landmark paper by Zöppritz et al. (1912) following the first author's untimely death in 1908.

As a result, when trying to relate energy and magnitude on a global scale, the authors compute an energy in the near field, from what are primarily measurements of accelerations, but a magnitude from ground motions (or perhaps estimates of velocity) obtained in the regional or far field. This is the source of the complex, occasionally arcane, nature of their derivations; in order to streamline the argument, we have relegated critical details of their calculations to the Appendix, which also presents a discussion of the (generally poor) robustness of the resulting parameters, including the slopes Q.

Table 1

Summary of parameters σ_i used by Gutenberg and Richter ([1942])

Parameter	Description	Defining equation	G–R value	Expected modern value	Robustness
σ_1	Slope of M versus $\log_{10}$ of displacement A	(21)	1		Forced
σ_2	Slope of M versus $\log_{10}$ of acceleration a	(22)	1.8	4 to 8	Very poor
σ_3	Inverse slope of M versus $\log_{10}$ of duration t_0	(23)	0.25	1/3 before initiation of saturation; grows thereafter	Poor
σ_4	Power law coefficient of kinetic energy E versus dominant period T_0 (at constant a_0)	(25)	2		Forced
σ_5	Power law coefficient of kinetic energy E versus acceleration a_0 (at constant T_0)	(25)	2		Forced
σ_6	Slope of $\log_{10}$ of dominant period T_0 versus magnitude M $\sigma_6 = \frac{1}{2} - \frac{1}{2\sigma_2}$	(29)	0.22	1/3 before initiation of saturation; grows thereafter	Poor
σ_7	Power law coefficient of dominant period T versus energy E $\sigma_7 = \sigma_6/Q$	(30)	0.12	1/3	Poor
σ_8	Power law coefficient of kinetic energy E versus acceleration a_0 (including effect of T_0) $\sigma_8 = 1 + \sigma_2(1 + \sigma_3)$	(A.2)	3.25	3 to 7, see Fig. 6	Very poor
Q	Slope of $\log_{10} E$ versus M in G–R relation $\boldsymbol{Q} = \boldsymbol{1} + \sigma_3 + \frac{1}{\sigma_2}$	(27)	1.8	1 before initiation of saturation	Poor

Table 2

Summary of parameters μ_j used by Gutenberg and Richter ([1956b])

Parameter	Description	Defining equation	G–R value	Expected modern value	Robustness
μ_1	Slope of m versus $\log_{10}$ of displacement-to-period ratio $q = \log_{10} A/T$	(31)	1		Forced
μ_2	Power law exponent of energy E versus duration t_0 (at constant q_0)	(32)	1		Forced
μ_3	Power law exponent of energy E versus logarithmic ratio q_0 (at constant t_0)	(32)	2		Forced
$\mu_4'; -\mu_4''$	Regression coefficients of q_0 versus local magnitude M_L and M_L^2	(33)	0.8; 0.01		
μ_4	μ_4' for a linear regression ($\mu_4'' = 0$). Also, slope of m versus M_L	(34)	0.7 (inferred)	1 before initiation of saturation	Poor
μ_5	Slope of $\log_{10}$ of dominant period T versus logarithmic ratio q	(36)	0.4		
μ_6	Slope of $\log_{10}$ of energy E versus magnitude m $\mu_6 = \mu_3 + \mu_5$	(38)	2.4		Poor
μ_7	Slope of "unified" magnitude m versus surface-wave magnitude M_s	(39)	0.63	1 before initiation of saturation; 2/3 and greater thereafter	Poor
Q	Slope of $\log_{10} E$ versus M in G–R relation $\boldsymbol{Q} = \mu_{\boldsymbol{6}} \cdot \mu_{\boldsymbol{7}}$	(40)	1.5	1 before initiation of saturation	Poor

4.1. The Derivation of $Q = 1.8$ by Gutenberg and Richter ([1942])

The derivation proposed in that paper is based on:

(i) The definition of magnitude M from the amplitude of ground motion A_0 recorded by a torsion instrument, which we can write schematically as

$$M = \sigma_1 \log_{10} A + c_1 = \sigma_1 \log_{10} A_0 + c'_1 \qquad (21)$$

with a slope σ_1 identically equal to 1, as imposed by Richter (1935); note that A in (21), being measured at a regional distance (typically up to a few hundred kilometers), is not necessarily an epicentral value (A_0), but is taken as such by the authors, the difference being absorbed into the constant c'_1.

(ii) The correlation (11) between magnitude and epicentral acceleration a_0 [their Eq. (20) p. 176] reproduced here as

$$M = \sigma_2 \log_{10} a_0 + c_2, \tag{22}$$

the slope $\sigma_2 = 1.8$ being obtained empirically from the comparison of magnitude values measured on torsion instruments in the regional field and accelerations derived from strong-motion seismograms in the vicinity of the epicenter;

(iii) An empirical relation between the duration of "strong ground shaking" t_0 and magnitude [their Eq. (28) p. 178]:

$$\log_{10} t_0 = \sigma_3 M + c_3, \tag{23}$$

where $\sigma_3 = 0.25$;

(iv) The calculation (10) of seismic energy E from the energy flux radiated vertically at the epicenter above a point source [their Eq. (24) p. 178], which we rewrite as

$$E = C_4 \cdot t_0 \cdot V_0^2, \tag{24}$$

where V_0 is ground velocity, and all c_i and C_i are constants independent of event size.

We have rewritten (10) as (24) to emphasize that (21), (22), and (24) involve *different physical quantities*, namely displacement [used in Richter's (1935) original definition], acceleration (available as strong-motion data in the epicentral area), and ground velocity (defining the kinetic energy flux). The authors' ensuing combinations of these equations through the use of the "period T_0" of the signal, presumably the dominant one, tacitly imply a harmonic character for the source, an additional complexity being that the concept of signal duration, t_0, is sensu stricto incompatible with this model, since a monochromatic signal is by definition of infinite duration.

Under that ad hoc assumption, they then derive their Eqs. (24) or (27) for E, which we rewrite as

$$\log_{10} E = c_4 + \log_{10} t_0 + \sigma_4 \log_{10} T_0 + \sigma_5 \log_{10} a_0 \tag{25}$$

with $\sigma_5 = 2$ identically [from (24), i.e., the power of 2 in the kinetic energy] and $\sigma_4 = 2$ identically (two powers of T_0 going from V_0^2 to a_0^2). Similarly, they transform (21) into

$$M = \sigma_1 \log_{10} a_0 + \sigma_4 \log_{10} T_0 + c_5, \tag{26}$$

equivalent to their Eq. (31). Substituting (22), (23), and (26) into (25) (note that σ_4, in principle equal to 2, and hence all *direct* reference to T_0, are eliminated), they obtain

$$\log_{10} E = QM + C_0 \tag{27}$$

with

$$Q = \sigma_1 + \sigma_3 + \frac{\sigma_5 - \sigma_1}{\sigma_2} = 1 + \sigma_3 + \frac{1}{\sigma_2} \approx 1.8. \tag{28}$$

The slope $Q = 1.8$ is thus explained as a combination of the various slopes σ_i. The latter are of a very different nature: $\sigma_1 = 1$ and $\sigma_5 = 2$ were fixed in Gutenberg and Richter's (1942) model, while σ_2 and σ_3 (1.8 and 0.25, respectively) were obtained empirically by the authors, and as such could vary, impacting significantly the value of Q derived from (28). We emphasize this point in the third member of Eq. (28), which leaves only σ_2 and σ_3 as variables.

As summarized in Table 1, the critical examination of this derivation detailed in the Appendix shows that the robustness of $Q = 1.8$ with respect to various assumptions underlying the computations is poor.

Note finally that Gutenberg and Richter (1942) combined their Eqs. (20) and (31) (our Eqs. 22 and 26) to obtain the dependence with magnitude of the dominant period T_0, which had dropped out of their derivation of Q:

$$\log_{10} T_0 = \sigma_6 M + c_6 \quad \text{with}$$
$$\sigma_6 = \frac{1}{\sigma_4} - \frac{\sigma_1}{\sigma_4 \cdot \sigma_2} = 0.22, \tag{29}$$

further leading to

$$\log_{10} T_0 = \sigma_7 \log_{10} E + c_7; \qquad \sigma_7 = \frac{\sigma_6}{Q} = 0.12,$$

$$(30)$$

some limitations of those two relations being further discussed in the Appendix.

4.2. The Final Regressions: the Road to $Q = 1.5$ (Gutenberg and Richter 1956b)

In this section, we paraphrase that paper to underscore the fundamental steps which led the authors to $Q = 1.5$. In the various equations relating E, m, A, etc., we will use formal slopes comparable to the σ_i used above, but with the notation μ_j to avoid confusion; γ_j will be constants substituting for the c_i above, largely irrelevant in the present discussion. Note that indexing of the parameters σ_i and μ_j is in both cases sequential in the derivation, so that σ and μ of identical index ($i = j$) may not describe comparable physical relations. The results are summarized in Table 2, in a format similar to that of Table 1; again, additional details can be found in the Appendix.

The situation differs from the previous derivation [Sect. 4.1 above, after Gutenberg and Richter (1942)] in several respects. First, and most importantly, the authors introduce the variable $q_0 = \log_{10}(A_0/T_0)$; following Gutenberg (1945c), the body-wave magnitudes, in particular m_B and hence m, are now defined from q_0, rather than from the logarithm of A_0:

$$m = \mu_1 \cdot q_0 + \gamma_1 = \mu_1(\log_{10} A_0 - \log_{10} T_0) + \gamma_1$$

$$(31)$$

with $\mu_1 = 1$ identically (Gutenberg and Richter 1956b; Eq. 11). Next, the authors now consider as their reference magnitude the unified magnitude m rather than Richter's (1935) original scale (restricted to local records of California earthquakes). Finally, several relationships now feature nonlinear (albeit weak) terms.

The fundamental difference between (31) and (21) stems from an implicit variation of the dominant period T_0 with earthquake size, which would be expected from scaling laws. However, the combined influence of anelastic attenuation and instrument response can act as a bandpass filter, reducing variations in the dominant period, which may then

significantly violate scaling laws, an effect enhanced at the short periods considered by the authors. For this reason, it may not be possible to define an expected value of the parameters μ under modern theories, and we leave blank several entries in the relevant column in Table 2.

Gutenberg and Richter's (1956b) next step is their Eq. (8) p. 10, equivalent to (24), and leading to

$$\log_{10} E = \gamma_2 + \mu_2 \log_{10} t_0 + \mu_3(\log_{10} A_0 - \log_{10} T_0)$$
$$= \gamma_2 + \mu_2 \log_{10} t_0 + \mu_3 q_0$$

$$(32)$$

with $\mu_2 = 1$ and $\mu_3 = 2$ identically. The introduction of q_0 rather than A_0 (with a nonconstant period T_0) then results in their new Eq. (9):

$$q_0 = \gamma_3 + \mu_4' M_L - \mu_4'' M_L^2 \qquad (33)$$

with $\mu_4' = 0.8$ and $\mu_4'' = 0.01$ (empirical values). Note that (33) expresses the relationship between m and M_L (Gutenberg and Richter 1956b; Eq. 14), and under the assumption that its curvature can be neglected, μ_4 is also the slope of m versus M_L:

$$m = \mu_4 M_L + \gamma_4. \qquad (34)$$

Note that, by comparison with (29), and assuming again that μ_4'' can be neglected, Eq. (33) leads to

$$\sigma_6 = \frac{1 - \mu_4}{\sigma_1} = 0.2. \qquad (35)$$

Next, the authors study empirically the relation between the duration t_0 in the epicentral area and the newly defined parameter q_0:

$$\log_{10} t_0 = \mu_5 q_0 + \gamma_5. \qquad (36)$$

They obtain a slope $\mu_5 = 0.4$ [their Eq. (10)], which we have confirmed (0.40 ± 0.06) by regressing the 19 points on their Fig. 6a [which are probably a subset of the previously mentioned dataset in Fig. 1 of Gutenberg and Richter (1956a)].

Note that the combination of (21), (23), (35), and (36) leads to

$$\mu_5 = \frac{\sigma_3 \sigma_1}{1 - \sigma_6} = \frac{\sigma_3}{\mu_4}, \qquad (37)$$

which takes the observed value $\mu_5 = 0.4$ for $\mu_4 = 0.8$ and $\sigma_3 = 0.32$ as favored by Gutenberg and Richter (1956a), but $\mu_5 = 0.31$ for $\sigma_3 = 0.25$ under the first

combination. Although not specifically spelt out in Gutenberg and Richter (1956b), the combination of (31), (32), and (37) then leads, for $\mu_5 = 0.4$, to

$$\log_{10} E = \mu_6 m + \gamma_6 \text{ with } \mu_6 = \frac{\mu_3 + \mu_2 \mu_5}{\mu_1} \quad (38)$$
$$= \mu_3 + \mu_5 = 2.4,$$

and then to (2), which as discussed in the Appendix, remains dependent on σ_3 through (37), and hence on the dataset in Fig. 1 of Gutenberg and Richter (1956a).

Using this slope of 2.4 in (2), and in order to finalize $Q = 1.5$ in (1), there remains to justify (3). That relation traces its origin to Fig. 9 of Gutenberg and Richter (1956a, p. 138), which examines the slope μ_7 regressing m versus M_s:

$$m = \mu_7 M_s + \gamma_7 \quad (39)$$

[note, however, that instead of m in (39), Gutenberg and Richter (1956b) apparently use the uncorrected body-wave magnitude (but with the notation M_B, as opposed to m_B)]. They describe an average slope of 0.4 between $(M_s - M_B)$ and M_s, equivalent to $\mu_7 = 0.6$, later revised to $\mu_7 = 0.63$ by Gutenberg and Richter (1956b). We have verified that a modern regression of the 105 points in their Fig. 9 does yield $1 - \mu_7 = 0.37 \pm 0.03$. Then, the slope Q is just the product

$$Q = \mu_6 \cdot \mu_7, \quad (40)$$

which takes the value 1.51, justifying (1).

4.3. Discussion

The detailed examination of the derivations of the slopes $Q = 1.8$ in Gutenberg and Richter (1942) and later $Q = 1.5$ inferred from Gutenberg and Richter (1956b), and the critical analysis of their underlying assumptions carried out in the Appendix, should severely limit the confidence of the modern reader in the resulting slopes Q and in particular in the value 1.5, proposed in their last contributions and now universally enshrined into seismological dogma. We recap here some of the most serious limitations documented in the present study:

- *First, it is not clear that they correctly measured the energy flux at the epicenter.*

As we have stated several times, the algorithm used by Gutenberg and Richter (1942) to evaluate energy flux near the epicenter consists of squaring the product of the maximum ground acceleration and the dominant period, and multiplying the result by the duration of sustained maximum ground motion. This is only a gross approximation to the integral defining energy flux, as defined later, e.g., by Haskell (1964), and suffers from an inherent flaw, the concept of a single (or dominant) frequency being incompatible with that of a finite signal duration.

- *Next, the authors were using a simplified analytical model.*

The latter essentially predated any knowledge of anelastic attenuation, for which models would start being available (albeit at much lower frequencies) in the late 1950s and early 1960s (Ewing and Press 1954; Satô 1958; Anderson and Archambeau 1964); of an adequate physical representation of the earthquake source as a double-couple (Knopoff and Gilbert 1959); or even of the concept of a spatially extended source, an idea pioneered by Lamb (1916) and heralded analytically by Ben-Menahem (1961). Feeling constrained to work in the near field, Gutenberg and Richter were using the model of a point source, whereas most earthquakes in their datasets would have had fault lengths comparable to, or greater than, their hypocentral depth. It would take six years following Gutenberg's death for Wu (1966) to regroup these later developments into a blueprint for the definitive computation of radiated energy from teleseismic datasets, which would be put to fruition only when digital data became available in the 1970s.

- *In addition, the robustness of their results is seriously cast in doubt.*

The final slopes Q are obtained from the parameters σ_i and μ_j, which relate observables of a high-frequency nature, strongly affected if not fully controlled by small-scale heterogeneity within the seismic source, as now universally documented by source tomography studies of recent megathrust events (Ishii et al. 2007; Lay et al. 2011). Such
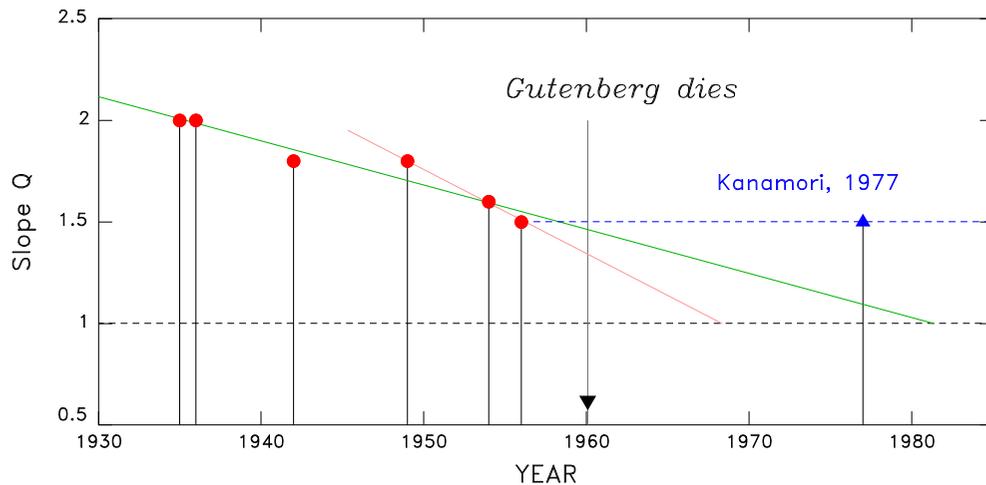
Figure 5
Evolution with time of the value of the slope Q in the various relations proposed by Gutenberg and Richter between seismic energy and magnitude. The dashed line is the value ($Q = 1$) derived theoretically for a point source double-couple. The vertical downpointing arrow indicates Gutenberg's death on 25 January 1960. The green line shows a linear regression through the six published values (1935–1956) and the pink one through the last three (1949–1956). The dashed blue line expresses the value $Q = 1.5$, frozen into perpetuity as the last one published before B. Gutenberg's death

source structure acts to defy the scaling laws which are inherent in the derivations of Q proposed by Gutenberg and Richter (1942, 1956b). Not surprisingly then, Eqs. (22) and (23) [or (31) and (36)] formalizing these scaling laws have not been upheld by modern strong-motion studies in the near field.

One can only speculate as to how Gutenberg and Richter may have pursued this line of work, but for the former's sudden death in 1960. Despite the promise of a full revision of magnitude scales (Gutenberg and Richter 1956a; abstract p. 165), no such work came forth during the next three years, a time when Gutenberg apparently shifted his main activity to studies in structural seismology, especially regarding the Earth's core (Gutenberg 1958). However, it is highly probable that he would have been, at the time of his death, cognizant of the new representation of the seismic source as a double-couple quantified by a seismic moment M_0, introduced in the West a year earlier by Knopoff and Gilbert (1959). One can only assume that, eventually, Gutenberg would have sought to relate magnitude to M_0, even though the first measurement of seismic moment from long-period waves had to wait until Aki's (1966) study of the 1964 Niigata earthquake, and a sufficient dataset of values until the early 1970s,

allowing the landmark studies by Kanamori and Anderson (1975) and Geller (1976).

Figure 5 summarizes the evolution of the slope factor Q in Gutenberg and Richter's papers from 1935 until Gutenberg's death. A further step in speculation would address how this inescapable, regular *decrease* would have continued, had the two authors been able to keep a proactive collaboration. A linear regression of the three points in the 1950s, when the authors' work was most supported analytically, predicts that they would have reached the more justifiable value $Q = 1$ in 1968. A full regression of the six values published since 1935 would suggest 1981. However, we note that, by then, Gutenberg would have been 92 years old.

5. Conclusion and Perspective

A detailed, occasionally forensic examination of the algorithms used by Gutenberg and Richter (1942; 1956b) suggests that they may have been legitimate under what was the state of the art of seismology in the 1940s and 1950s, but are nowadays difficult to reconcile with modern advances in source theory and Earth structure, including anelasticity, as well as with

the advent of high-frequency digital data. In this context, the physical models underlying the derivation of the successive values of the slope Q, including the empirical relations used by Gutenberg and Richter, can now appear simplistic if not outright inaccurate, e.g., the model of a point source, and the implicit assumption of a monochromatic signal. As a result, the derived values of Q clearly lack robustness, and in particular the last one (1.5) hardly deserves the apparently sacred character which led to its being enshrined into the definition of moment magnitude M_w by Kanamori (1977).

We recall that, in their last paper on the subject, Gutenberg and Richter (1956a) commented that there may not necessarily be a single relationship between seismic energy and any magnitude scale. In retrospect, this statement appears visionary, as we now understand that the concept of radiated energy is fundamentally anchored in the high-frequency part of the source spectrum, whereas individual magnitude scales target specific periods which can belong to significantly different parts of the spectrum. Note that Kanamori (1983) echoed this statement, asserting that "It is impossible to represent all [...] parameters by a single number, the magnitude."

Since the advent of digital seismology in the late 1970s, it has become clear that many seismic sources do indeed follow laws of similitude allowing the description of many, if not all, of an event's source properties on the basis of a single number, namely its seismic moment M_0, which Kanamori (1977) has proposed to cast into the "magnitude" M_w, thus introducing a parameter combining the rigor of the underlying quantification of a physical quantity with the practicality of empirically derived magnitude scales. Such source properties generally include radiated energy E, as documented by extensive catalogs such as Choy and Boatwright's (1995).

At the same time, a number of violations to these scaling laws have been regularly documented, both in the form of "slow" events, whose red-shifted spectrum leads to a deficiency in radiated energy, and of "snappy" ones featuring a blue-shifted spectrum. Such rogue events bear crucial societal relevance, since the former include the so-called tsunami earthquakes (Kanamori 1972), whose real-time identification remains a challenge in operational tsunami warning, while the latter have proven ominously destructive due to exceptional levels of ground acceleration, e.g., reaching 2.2g during the relatively small $(M_w = 6.2)$ 2011 Christchurch earthquake (Holden 2011).

However, little progress has been made in the understanding of parameters possibly controlling their occurrence. For example, it is not clear whether all megaquakes (defined as having moments greater than 10^{29} dyn cm) feature source slowness (Okal 2013, Fig. 10), or whether there exists a regional trend controlled by simple tectonic patterns to the distribution (and hence to the predictability) of tsunami earthquakes, an idea hinted at by Okal and Newman (2001), and further supported by the 2006 Java and 2012 El Salvador events. In this context, the availability of catalogs asserting a diversity of source parameters is of crucial necessity for further research into such topics.

Yet, such catalogs are not (or no longer) made readily available. For example, an updated extension of Choy and Boatwright's (1995) extremely valuable dataset is not widely distributed, and to our best knowledge, their dataset is no longer being routinely updated. Similarly, for many years, the National Earthquake Information Center of the United States Geological Survey had distributed an electronic catalog of epicenters, listing for each event a set of *various* and occasionally different conventional magnitudes (m_b, M_s, M_L), in addition to moment estimates transcribed as M_w. This allowed the easy identification of anomalous events, e.g., through a simple search for an $m_b : M_s$ disparity (a predecessor to the E/M_0 ratio in the pre-digital age). As of 2015, this practice has been discontinued and replaced by the issuance of a single magnitude, whose nature (body, surface, local, moment, etc.) is not clearly specified, with the probable goal of making the catalog less confusing to the lay user, admittedly a legitimate concern. As a result, the individual researcher has lost a powerful tool to further our understanding of earthquake source properties. Within a few decades, the careless or simply uninformed investigator might be tempted to conclude that earthquakes violating scaling laws ceased to occur after 2014.

Incidentally, seismology is not alone in featuring a diversity of source properties acting to restrict the range of validity of scaling laws. For example, the size of hurricanes has been described through a Safir–Simpson "category" index, a single number quantifying several of their properties, such as geometrical extent, maximum wind velocity, and underpressure at the center of the eye; as such, it constitutes a measure conceptually similar to an earthquake magnitude. However, major hurricanes have shown that such an approach constitutes a drastic simplification, with Sandy in 2012 featuring a diameter of 1800 km for a minimum pressure of 940 mbar and winds of 185 km/h (Blake et al. 2013), while the 2015 Mexican hurricane Patricia registered a record low pressure of 872 mbar and winds of 345 km/h concentrated in a system not more than 800 km across (Kimberlain et al. 2016). Such limitations for hurricane scaling laws are not intrinsically different from the diversity expressed for seismic sources through variations in energy-to-moment ratios, and similarly prevent an accurate societal prediction of the level of natural hazard based on a single quantifier (magnitude or category) of the event. Another example would be the Volcanic Explosivity Index (Newhall and Self 1982), widely used to rank large volcanic eruptions, but ignoring specific properties significantly affecting their dynamics as well as their climatic and societal effects (Miles et al. 2004).

In this context, it behooves the seismological community to observe, respect, and fully document the occasional diversity in source properties of earthquakes, notably by restoring the now abandoned practice of systematic reporting of radiated seismic energy. This is a necessary step towards the furtherance of our understanding of seismic source properties in the context of their societal impact, in keeping with the pioneering work of Beno Gutenberg and Charles Richter.

Appendix: Critical Details of Gutenberg and Richter's Derivations

In this Appendix, we present some critical details of the derivations of the slopes $Q = 1.8$ (Gutenberg and Richter 1942) and $Q = 1.5$ (Gutenberg and Richter 1956b), and in particular examine their robustness which eventually controls that of the inferred parameters Q.

A.1. Derivation of $Q = 1.8$ (Gutenberg and Richter 1942), Eqs. (21–30) of Main Text

Note that any increase in the slope σ_2 in (28) will lead to a decrease in Q. In the context of modern strong-motion seismology, it has become clear that an equation of the form (22) may not apply, and certainly not universally, between local magnitude and the logarithm of peak acceleration, with most modern models featuring nonlinear relationships (e.g., Abrahamson and Silva 1997). In practice, and as illustrated, e.g., by Bolt and Abrahamson (2003; their Fig. 3), a value of $\sigma_2 \approx 4$ may be legitimate around $M_L = 6$, but would grow as high as $\sigma_2 \approx 8$ at $M_L = 7$, illustrating the well-known effect of saturation of maximum acceleration with moment (Anderson and Lie 1994). In this context, it is difficult to justify on theoretical grounds any form of Eq. (22), let alone the value $\sigma_2 = 1.8$, which appears at any rate as a lower bound of values that could be derived in narrow ranges of magnitudes from modern datasets. In hindsight, σ_2 probably constitutes the least well constrained among the slopes σ_i in Gutenberg and Richter's (1942) approach.

As for the slope σ_3, it expresses the variation with M of the duration parameter t_0 defined by Gutenberg and Richter (1942) as the time over which "equal sinusoidal waves" are recorded at a near-field receiver. From a modern theoretical standpoint, t_0 would be expected to scale with the total duration of

rupture on the fault, and hence as $M_0^{1/3}$. Assuming that the magnitude M is taken in its range of initial saturation (featuring a slope of 2/3 with $\log_{10} M_0$), σ_3 should then equal 1/2 (Trifunac and Brady 1975). By contrast, Gutenberg and Richter (1942) obtained empirically $\sigma_3 = 1/4$. Gutenberg and Richter (1956a) later revised this estimate to 0.32, based on a dataset of 29 measurements (their Fig. 1, p. 110), for which a modern regression yields a slope of 0.30 ± 0.02, or 0.29 ± 0.03 when excluding a lone measurement for a barely detectable shock (Richter and Nordquist 1948); at any rate, we have found that the quality of fit improves by only 5 % between slopes of 0.25 and 0.30.

Gutenberg and Richter's (1942) parameter t_0 may be comparable to the duration s defined in strong-motion seismology; in that framework, we note indeed a relationship of the form (23) between $\log_{10} s$ and M, with $\sigma_3 = 0.32$, quoted by several authors (e.g., Esteva and Rosenblueth 1964). However, further work has shown a poor dependence of $\log_{10} s$ on magnitude, and suggested a better fit using a linear relation between s and M (Housner 1965; Vanmarcke and Lai 1980). Various authors, including Housner (1965) and Trifunac and Brady (1975), have reported a *decrease* of duration with Mercalli intensity, especially when the former is measured using acceleration spectra, which for large earthquakes are expected to be strongly red-shifted; this led Housner (1965) to define a limit of "maximum duration" growing, perhaps linearly, albeit rather erratically, with magnitude. The bottom line of this discussion is that an equation of the form (23) is poorly fit by modern strong-motion data, and hence that the parameter σ_3, if at all justifiable, is prone to substantial variations, which in turn will impact the slope Q.

Note finally that the duration of strong motion, t_0, should not be confused with the coda duration upon which so-called duration magnitudes have been proposed, which incidentally also feature a broadly variable range of slope parameters σ (Lee and Stewart 1981).

In addition to these uncertainties in the values of σ_2 and σ_3 in (28), we stress, once again, that a fundamental difficulty with Gutenberg and Richter's (1942) proposed derivation remains that it assumes a harmonic oscillation in order to relate the displacement A_0 used in the definition of magnitude to the velocity V_0 inherent in the calculation of the energy flux. In particular, the fact that the period T_0 required to relate them disappears from the final Eq. (28) should not obscure that its implicit existence underlies the derivation.

An additional, and significant, problem stems from the assumption of a spherical wave front at the epicenter above a point source, for the calculation of the energy flux. There are two issues there which are bound to fail for large earthquakes whose fault dimension, L, becomes larger than the source depth. First, in the near field, ground motion is essentially proportional to the slip on the fault, and as such grows only like $M_0^{1/3}$, a result well known to practitioners of deformation codes such as Mansinha and Smylie's (1971) or Okada's (1985). Second, Gutenberg and Richter (1942) interpret the energy computed at an epicentral station as an energy flux, which they then integrate over a focal sphere whose radius is the hypocentral depth, thus clearly assuming a point source. By contrast, modern calculations of energy flux are carried out in the far field, many fault lengths away from the source, and as such are immune to that problem (e.g., Boatwright and Choy 1986).

More generally, we note that Gutenberg and Richter (1942) had appropriately recognized the time finiteness of the source by introducing its duration t_0 at the epicenter, and even its dominant period T_0, which inherently requires a finite source time, but they never addressed the problem of the *spatial* finiteness of the source, and their models always considered a point source in space. Only in the last section of their last paper (Gutenberg and Richter 1956a; p.142) would they recognize a future need to consider this problem, which was to be fully studied only after Gutenberg's death (Ben-Menahem 1961).

Gutenberg and Richter (1942; Eq. (41) p. 188) also sought to directly relate the acceleration at the epicenter, a_0, to the energy E of the earthquake. Eliminating M between (22) and (27) leads to

$$\log_{10} E = \sigma_8 \log_{10} a_0 + c_8 \qquad (A.1)$$

with

$$\sigma_8 = \sigma_5 - \sigma_1 + \sigma_2(\sigma_1 + \sigma_3) = 1 + \sigma_2(1 + \sigma_3)$$
$$\approx 3.25; \quad \frac{1}{\sigma_8} = 0.31.$$

(A.2)

This inverse slope being remarkably close to 1/3, the authors rounded it, and proposed in their Eq. (42) a cubic root dependence of acceleration a_0 on energy E. The modern reader should resist the temptation to associate this result to the growth of epicentral displacement A_0, itself controlled by seismic slip Δu (at least for sources shallower than their spatial extent), as $M_0^{1/3}$: for point source double-couples, Newton's (1687) law does predict a_0 directly proportional to M_0, but for finite sources, scaling laws would actually predict a corner frequency scaling as $M_0^{-1/3}$, leading to a *decrease* of peak acceleration as $M_0^{-1/3}$, a result generally in line with the empirically reported saturation of peak acceleration with earthquake size (Anderson and Lie 1994). We believe that the value $\sigma_8 \approx 3$ is a fortuitous result of the particular choice of slopes σ_2 and σ_3. For example, the "second" combination of $\sigma_2 = 4$ [as suggested by Bolt and Abrahamson (2003) for moderate earthquakes] and $\sigma_3 = 0.32$ [as used later by Gutenberg and Richter (1956a)] does lead to $Q = 1.57$, close to the values of 1.6 suggested by Gutenberg and Richter (1954) and 1.5 as given by (1), but this second combination nearly doubles σ_8, to a value of 6.28.

We next consider the slopes σ_6 and σ_7, defined in (29) and (30). As mentioned above, these slopes are difficult to reconcile with modern scaling laws, which would predict $\sigma_8 = 1/3$ under the assumption that T_0 is directly related to the inverse of a source corner frequency. We note, again, that σ_7 will vary strongly with the coefficients σ_2 and σ_3 determined empirically by Gutenberg and Richter (1942, 1956a). For example, for ($\sigma_2 = 4$; $\sigma_3 = 0.32$), σ_6 rises to 0.38 and σ_7 doubles, to 0.24.

More systematically, we explore in Fig. 6 the variation of the slopes Q, σ_8 (through its physically more meaningful inverse, $1/\sigma_8$), and σ_7 as a function of the empirical slopes σ_2 and σ_3. The former is allowed to vary between 0 and 6, covering the range suggested by strong-motion investigations (Bolt and Abrahamson 2003), and the latter between 0.1 and 0.6, to include the theoretical value (0.5) expected

from scaling laws under the assumption that t_0 represents the inverse of a corner frequency of the source spectrum. The resulting values of Q, $1/\sigma_8$, and σ_7 are simply computed from (27), (29), (30), and (A.2) under the constraints $\sigma_1 = 1$, $\sigma_4 = 2$, and $\sigma_5 = 2$, and their contours are color-coded using individual palettes. The bull's eyes indicate the "first" combination ($\sigma_2 = 1.8$; $\sigma_3 = 0.25$) used by Gutenberg and Richter (1942), and the triangles the second combination [larger values of $\sigma_2 = 4$ suggested by strong-motion data; and $\sigma_3 = 0.32$ later adopted by Gutenberg and Richter (1956a)]. Figure 6 provides a quantitative illustration of the absence of precision of the slopes computed by Gutenberg and Richter, most notably of their energy-to-magnitude parameter Q. The last column of Table 1 summarizes their lack of robustness.

A.2. The Final Regression: $Q = 1.5$ (Gutenberg and Richter 1956b), Eqs. (31–40) of Main Text

First, we note that this final study reaches an extreme complexity in the definition of various magnitudes, considering no fewer than seven such scales. They are (1) Gutenberg's (1935) original one for locally recorded California shocks, appropriately renamed a local magnitude M_L; (2) the surface-wave magnitude M_s^G introduced by Gutenberg and Richter (1936) and formalized by Gutenberg (1945a); (3) the teleseismic body-wave magnitude m_B defined by Gutenberg (1945b, c); (4) its value corrected to M_B to make it more compatible with M_s^G; (5) conversely, an m_s applying the opposite correction to M_s^G in order to make it compatible with m_B; (6) a unified magnitude M (without subscript), consisting of a "weighted mean" between M_B and M_s^G (but with no details provided about the weighting process); and (7) a corresponding, "final" weighted mean m (without subscript) between m_B and m_s. We have used the temporary notation M_s^G to emphasize that this magnitude (2) is not a priori equivalent to the Prague M_s later defined by Vaněk et al. (1962), since the former measures an amplitude A and the latter the ratio A/T of amplitude to period; however, because of the prominence of 20-s waves in standard long-period teleseismic records, the two approaches are largely compatible; in addition, we note the similarity in
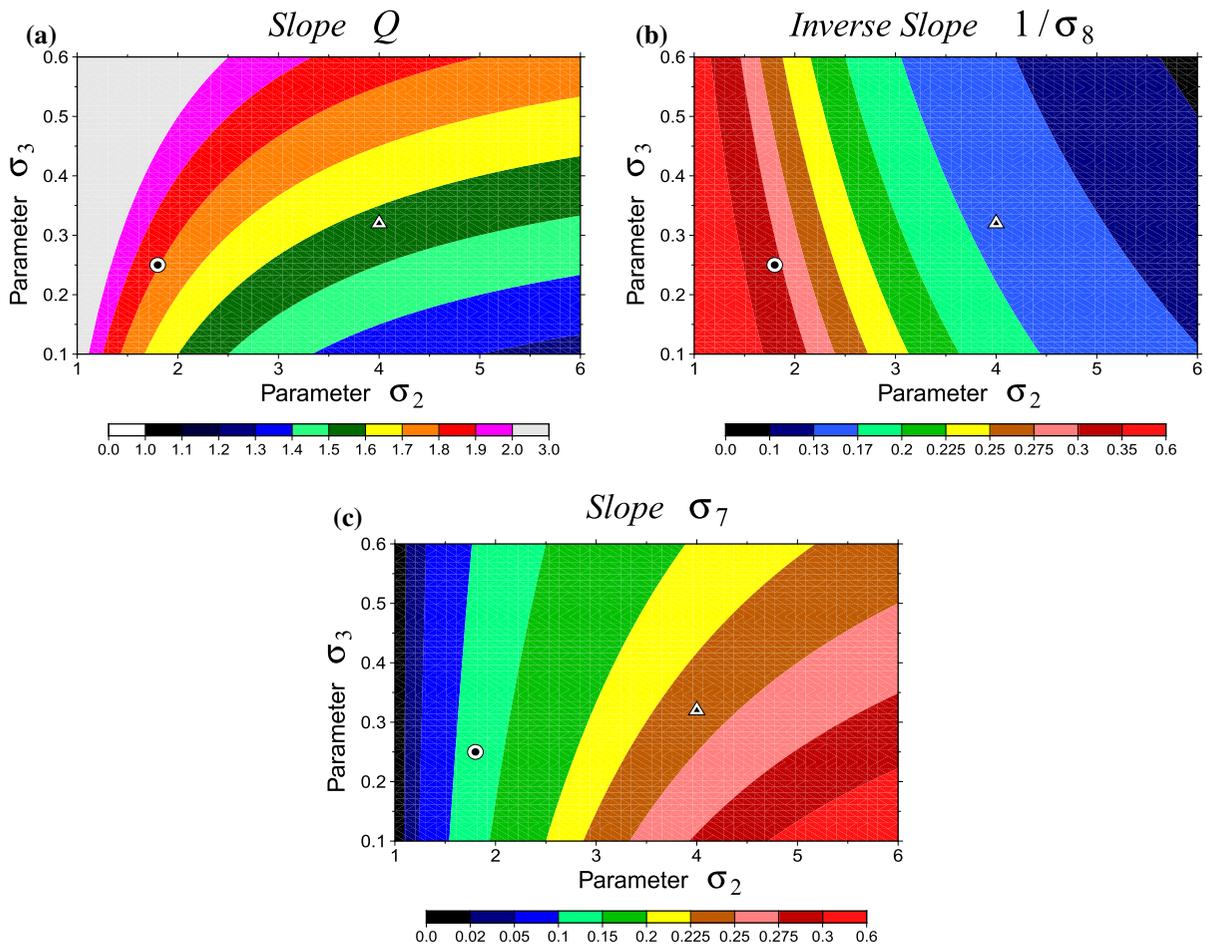
Figure 6
Variation of **a** slope Q defined in (1), **b** inverse slope $1/\sigma_8$ defined in (A.1), and **c** slope σ_7 defined in (30), as a function of the empirical slope parameters σ_2 (in abscissa) and σ_3 (in ordinate). Each frame uses a different palette, reproduced at its bottom. The bull's eye symbols refer to the first combination ($\sigma_2 = 1.8$; $\sigma_3 = 0.25$), used by Gutenberg and Richter (1942), the centered triangles to the second combination [$\sigma_2 = 4$ as suggested by modern strong-motion studies; $\sigma_3 = 0.32$ as used by Gutenberg and Richter (1956a)]

distance corrections between M_s^G [$1.656 \log_{10} \Delta$ as opposed to $2 \log_{10} \Delta$ in Gutenberg and Richter (1936)] and the Prague M_s ($1.66 \log_{10} \Delta$), even though the Prague group did consider many previously proposed values of the slope (G. Purcaru, pers. comm., 1988), ranging from 1.31 (Nagamune and Seki 1958) to 1.92 (Bonelli and Esteban 1954). As a result, the two scales may indeed be essentially equivalent, allowing us to henceforth drop the superscript "G."

Among the critical issues regarding the derivation of $Q = 1.5$ by Gutenberg and Richter (1956b), we first address the effect of using (33) on the variable σ_6

defined in Sect. 4.1, and characterizing the variation of the dominant period T_0 with magnitude. By comparison with (29), and assuming μ_4'' can be neglected, Eq. (33) leads to

$$\sigma_6 = \frac{1 - \mu_4}{\sigma_1} = 0.2, \qquad (A.3)$$

in good agreement with $\sigma_6 = 0.22$ as computed in Sect. 4.1 with the first combination ($\sigma_2 = 1.8$; $\sigma_3 = 0.25$) of parameters used by Gutenberg and Richter (1942). However, Eq. (33) regresses linearly with a slope $\mu_4 = 0.7$ between magnitudes of 1 and 9, but 0.66 between 5.5 and 8.5. The
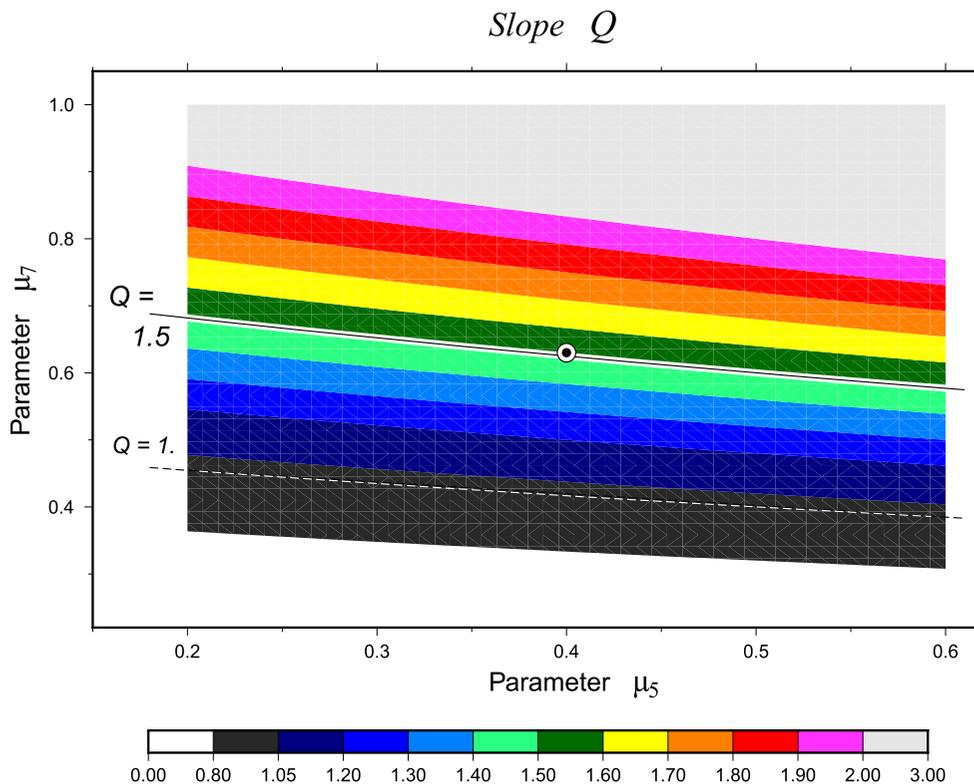
Figure 7

Variation of slope Q as computed in Gutenberg and Richter (1956b), as a function of the slopes μ_5 and μ_7 defined in (36) and (38). The bull's eye symbol identifies the parameters used by the authors, and the solid line the locus of other combinations yielding $Q = 1.5$. For reference, the dashed line similarly identifies the locus of $Q = 1$

corresponding values for σ_6 would be significantly altered, to 0.30 and 0.34, respectively.

We then consider the parameter μ_5 defined in (36), which takes the value 0.4 for $\mu_4 = 0.8$ and $\sigma_3 = 0.32$, as proposed by Gutenberg and Richter (1956a). We find that μ_5 can range from 0.31 under the first combination to as much as 0.48 for $(\sigma_3 = 0.32; \mu_4 = 0.66)$. Finally, excluding the lone datum for the barely detectable shock (Richter and Nordquist 1948) reduces the regressed slope μ_5 to the intermediate value 0.36 ± 0.09.

Next, we focus on Eq. (38), defining $\mu_6 = 2.4$, perhaps the most crucial step on the way to $Q = 1.5$. With μ_3 identically equal to 2, μ_6 is controlled by μ_5, itself the ratio of σ_3 to μ_4 (37). While Fig. 1 of Gutenberg and Richter (1956a) suggests an acceptable fit of (2) to a dataset of 21 points, their formal regression yields a higher slope ($\mu_6 = 2.57 \pm 0.19$), whose error bar includes the value 2.4, but which is

strongly controlled by Richter and Nordquist's (1948) minimal datum mentioned above, and by a nuclear test at Bikini, which clearly does not belong in a dataset examining the properties of earthquake sources. A regression of the remaining 19 points results in a greater, poorly constrained slope ($\mu_6 = 3.00 \pm 0.33$).

As summarized in Table 2, we conclude that, as in the case of their earlier calculation ($Q = 1.8$), the slope $Q = 1.5$ obtained by Gutenberg and Richter (1956b) is not robust when considering the various assumptions underlying it in the context of modern seismological theory.

A.3. Discussion and Conclusion

Given the empirical nature of the slopes σ_2 (relating magnitude to acceleration) and σ_3 (magnitude and duration), it would be in principle possible

to read Fig. 6 backwards, i.e., to investigate which values of σ_2 and σ_3 would be necessary to obtain the parameter $Q = 1.5$ proposed by Gutenberg and Richter (1956b) (we recall that the slopes σ_1, σ_4, and σ_5 must have their fixed values of 1, 2, and 2, respectively). We are motivated by the fact that the slope σ_2 for example, relating acceleration to magnitude, is documented from modern studies to be very poorly constrained and indeed could be very high; the "duration" slope σ_3 is also poorly constrained by strong-motion studies.

In this context, some constraints could, at least in principle, come from the parameters σ_8 and σ_7, which may be expected to take predictable values under seismic laws. But we have seen that the definition of σ_8 runs into the problem of saturation of acceleration with earthquake size (σ_8 could even be negative); only the slope σ_7 relating the (dominant) period T_0 to energy bears some legitimate hope of obeying scaling laws (with $\sigma_7 = 1/3$), since it relates two quantities of a fundamentally physical nature, which would be expected to scale predictably with earthquake size. Figure 6c shows that $\sigma_7 = 1/3$ will require extreme values of both σ_2 (greater than 5) and σ_3 (less than 0.2), clearly very different from those used by Gutenberg and Richter (1956a), but perhaps not impossible; incidentally, they will result in $Q \leq 1.4$.

However, the above discussion has shown that the various quantities measured by Gutenberg and Richter are all of a high-frequency nature (if for no other reason, because of the instruments they used in the near field), and thus sensitive to effects such as the fine structure of the source [e.g., large hetero-geneities of slip on the fault plane especially for the largest sources (Lay et al. 2011)]; as a result, the corresponding slopes, such as σ_7, may depart signif-icantly from their theoretical values. In conclusion, the derivation of $Q = 1.8$ proposed by Gutenberg and Richter (1942), and corrected to 1.6 by Gutenberg and Richter (1956a), stems from a particular choice of critical slopes (σ_2, σ_3), which may represent best fits obtained empirically from then available datasets, but which are neither robust, given now available strong-motion datasets, nor justifiable theoretically under the canons of modern source theory.

A similar approach could be attempted with the slopes μ_5 and μ_7, in search of constraints on $Q = 1.5$

as derived by Gutenberg and Richter (1956b). Figure 7 examines systematically the dependence of Q on those empirical parameters. Our discussion in Sect. 4.2 has shown that $\mu_5 = 0.4$ could be either under- or overestimated, with values between 0.5 and 0.3 being plausible. As for μ_7, while the value 0.63, reported by Gutenberg and Richter (1956b), is in excellent agreement with the slope of 2/3 expected under the first stage of saturation, where m_B (but not yet M_s) would have started to feel the effects of source finiteness, Fig. 8 predicts that the slope 2/3 should apply only at low magnitudes ($M_s < 5$), with significantly lower values expected for larger earth-quakes. A combination of $\mu_5 = 0.3$ and $\mu_7 = 0.5$ would yield $Q = 1.15$. Also, Gutenberg and Richter (1956a) note that a particular operational algorithm applies to measurements of the 1952 Kern County aftershocks; when those are excluded from the dataset in their Fig. 9, the slope μ_7 falls to 0.57 and Q to 1.37 (keeping $\mu_5 = 0.4$).

Finally, we note that, even though the authors had introduced nonlinear terms [e.g., in (35)], the
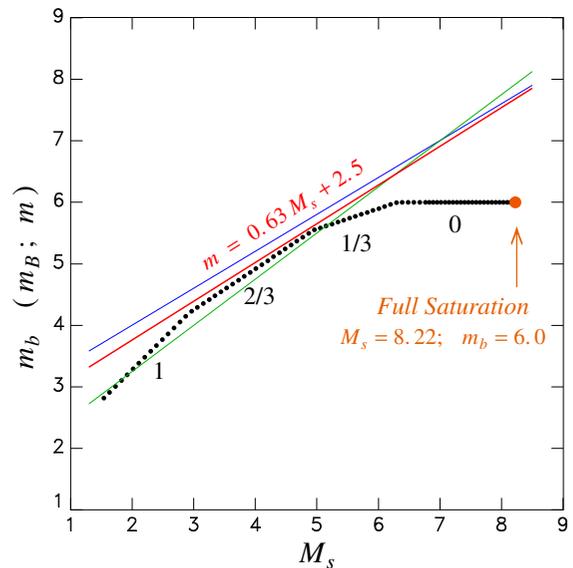

Figure 8
Variation of m_b as a function of M_s as predicted from a combination of (19) and (20) (solid dots). The theoretical slopes are indicated in black for each regime of partial or total saturation. Superimposed in red is the best fit (38) obtained for m versus M_s by Gutenberg and Richter (1956b), in blue the earlier regression by Gutenberg and Richter (1956a), and in green their fit to the 1952 Kern County aftershocks

derivation of $Q = 1.5$ ignores them. In this context, it is interesting to note that Gutenberg and Richter (1956a; p. 133) report that their colleague V.H. Benioff had suggested interpreting the curvature expressed in many of their graphs (best illustrated by their Fig. 3) as consisting of "two straight lines intersecting near magnitudes 5 and 6 [sic]," a visionary remark in view of the later description of staggered steps in magnitude saturation (Brune and Engen 1969; Geller 1976). Rather, Gutenberg and Richter elected to keep a quadratic form for (33), and then ignored the nonlinear terms in their derivation of $Q = 1.5$ (Gutenberg and Richter 1956b).

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