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70.

A Theoretical Discussion of Time Domain Magnitudes: The Prague Formula for M_s and the Mantle Magnitude M_m

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A combination of surface wave theory and phase-stationary asymptotics is used to relate the time domain amplitude $a(t)$ of a strongly dispersed wave and the moment M_0 of the source. This approximation is valid at sufficiently large distances, over 10° for 20-s Rayleigh waves. We apply this formalism to justify theoretically the Prague formula for M_s. Assuming a Rayleigh Q of 297, we can successfully model the theoretical distance correction as $1.66 \log_{10} \Delta$ in the range 20–160°. We also predict a relation of the form $\log_{10} M_0 = M_s + 19.46$, in good agreement with reported empirical values. Finally, we show that the theory requires M_s to be described by the product (aT); the use of the ratio (a/T) is a partial and ad hoc compensation for a large number of frequency-dependent terms ignored in the Prague formula. The same formalism can be applied to the inversely dispersed branch of mantle Rayleigh waves, between periods of 60 and 230 s. We provide the theoretical justification for the use of time domain measurements to obtain a mantle magnitude, and in particular for the modeling of the ratio a/A of the time domain and spectral amplitudes as $2/T$, at distances ranging from 20 to 120°. At greater distances, and in particular for multiple passages, an additional distance correction must be effected.

1. INTRODUCTION

With the development of digital data and progress in inversion theory, most earthquake sources are now described through their seismic moment M_0, which is given routinely by a variety of agencies. However, by far the most popular measure of earthquake "size" remains the 20-s surface wave magnitude M_s. In particular, *Geller and Kanamori* [1977] have shown that it provides continuity with historical events, since it is basically equivalent to the "Richter" magnitudes as listed, in particular, by *Gutenberg and Richter* [1954]. The International Association of Seismology and Physics of the Earth's Interior has adopted in 1961 the so-called Prague formula for M_s [*Vaněk et al.*, 1962]:

$$M_s = \log_{10} \frac{a}{T} + 1.66 \log_{10} \Delta + 3.3 \tag{1}$$

where a is the zero-to-peak amplitude of ground displacement measured in microns at a period T close to 20 s and Δ is the epicentral distance in degrees. However, there does not exist, to our knowledge, any full justification of this formula and of its theoretical relation to the seismic moment M_0. In particular, three points deserve critical analysis: (1) the systematic investigation of the theoretical basis and of the performance of the distance correction coefficient, 1.66; (2) the justification of empirical relations found in the literature between M_s and the seismic moment M_0, based on the use of the constant 3.3 in (1); and (3) finally, the practice of using the ratio (a/T) in (1).

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In this paper we use the classic theory of surface wave excitation and propagation to shed some light on these questions and in particular to discuss the theoretical range of application and limitations of the Prague formula. In addition, we discuss the extension of the concept of a time domain measurement of earthquake magnitude to much longer periods, in the framework of our companion investigation of mantle magnitudes [*Okal and Talandier*, this issue].

2. THEORY

The theory of the excitation of surface waves by an earthquake source has long been worked out by a number of investigators, notably, *Harkrider* [1964] and *Gilbert* [1970]. These studies have all been carried out in the frequency domain. In order to relate directly the amplitude of oscillation $x(t)$ observed in the time domain to the spectral amplitude $A(\omega)$, we consider the inverse Fourier equation

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega) e^{i\phi_0(\omega)} e^{i\omega t} e^{-ik(\omega)D} \, d\omega \tag{2}$$

for a particular group time t_0 and at an epicentral distance D. We further assume that the wave is strongly dispersed, i.e., that we can define a single (positive) frequency ω_0 such that the group velocity $U_0 = (d\omega/dk)_{\omega_0}$ equals exactly D/t_0. In addition, we assume that the spectral amplitude $A(\omega)$ and initial phase $\phi_0(\omega)$ are slowly varying in the vicinity of $\omega = \pm\,\omega_0$.

We proceed with a variation of the phase-stationary approximation, which we describe in some detail, keeping track of the various approximations in order to assess their applicability. We first concentrate on the positive frequency ω_0 and replace (2) by

$$x(t_0) = \frac{A(\omega_0)\, e^{i\phi_0(\omega_0)}}{2\pi} \int_{-\infty}^{\infty} e^{i(\omega t_0 - k(\omega)D)}\, d\omega \quad (3)$$

This approximation will be valid if $A(\omega)$ and $\phi_0(\omega)$ vary with frequency slower than do the phase terms kept under the integral. The validity of this approximation will be discussed later.

We consider a record band-pass-filtered in the range $\{\omega_{min}, \omega_{max}\}$. In practice, we will define these bounds as the two Airy frequencies limiting the interval of strong dispersion of the wave. We further write the integral in (3) in terms of the wave vector k:

$$I = \int_{k_{min}}^{k_{max}} \exp\left[i(\omega(k)t_0 - kD)\right] \cdot U(k) \cdot dk \quad (4)$$

and expand

$$\omega(k) = \omega_0 + U_0(k - k_0) + \frac{1}{2}\left[\frac{dU}{dk}\right]_{k_0} (k - k_0)^2 \quad (5)$$

Again, this approximation would not be valid if dU/dk vanished, for example, at the Airy phase observed for oceanic dispersion around 40 s. In (4), t_0 is large, and $U(k)$ is slowly variable, leading to

$$I = U_0\, e^{i(\omega_0 t_0 - U_0 k_0 t_0)} \times \int_{k_{min}}^{k_{max}} \exp\left[\frac{i}{2}\left[\frac{dU}{dk}\right]_{k_0} t_0 (k - k_0)^2\right] \cdot dk \quad (6)$$

The quantity $(dU/dk)_{k_0}$ is simply related to the slope $\alpha = dU/dT$ of the group velocity as a function of period along the dispersed branch:

$$\left[\frac{dU}{dk}\right]_{k_0} = -\alpha \frac{2\pi}{\omega_0^2} U_0 \quad (7)$$

and (6) simply becomes

$$I = U_0\, e^{i(\omega_0 t_0 - U_0 k_0 t_0)} \frac{1}{2}\frac{\omega_0}{\sqrt{\alpha D}}\, e^{-i\pi/4} \times \left[\operatorname{erf}\left[e^{i\pi/4}\frac{\sqrt{\pi\alpha D}}{\omega_0}(k_0 - k_{min})\right] + \operatorname{erf}\left[e^{i\pi/4}\frac{\sqrt{\pi\alpha D}}{\omega_0}(k_{max} - k_0)\right]\right] \quad (8)$$

Similarly, the contribution of the negative frequency, $-\omega_0$, is the complex conjugate

$$I^* = U_0\, e^{-i(\omega_0 t_0 - U_0 k_0 t_0)} \frac{1}{2}\frac{\omega_0}{\sqrt{\alpha D}}\, e^{i\pi/4} \times \left[\operatorname{erf}^*\left[e^{i\pi/4}\frac{\sqrt{\pi\alpha D}}{\omega_0}(k_0 - k_{min})\right] + \operatorname{erf}^*\left[e^{i\pi/4}\frac{\sqrt{\pi\alpha D}}{\omega_0}(k_{max} - k_0)\right]\right] \quad (9)$$

There are no additional approximations between (6) and (8) and (9). As t_0 is changed by dt, the phase of the exponential terms varies as

$$\omega_0\, dt + [t_0\, d\omega - U_0 t_0\, dk] - [k_0 t_0\, dU + k_0 U_0\, dt] \quad (10)$$

The last two brackets vanish (remember $U\, t = D$ constant), which gives the resulting signal the appearance of a sinusoid of angular frequency ω_0. Its zero-to-peak amplitude is simply

$$a = \frac{A(\omega_0)\, U_0\, \omega_0}{2\pi\sqrt{\alpha D}} \times \left|\operatorname{erf}\left[e^{i\pi/4}\frac{\sqrt{\pi\alpha D}}{\omega_0}(k_0 - k_{min})\right] + \operatorname{erf}\left[e^{i\pi/4}\frac{\sqrt{\pi\alpha D}}{\omega_0}(k_{max} - k_0)\right]\right| \quad (11)$$

This expression is clearly controlled by the behavior of the function $\xi(x) = \operatorname{erf}(e^{i\pi/4}x)$ for real values of x. For small values it behaves as $(2/\sqrt{\pi})e^{i\pi/4}x$ and approaches the real number 1 for $x \to \infty$.

3. APPLICATION TO M_s

In the case of the magnitude scale M_s we consider the dispersion of Rayleigh waves for an average Earth model (which we obtained by combining *Oliver*'s [1962] oceanic and continental models with weighting ratios of 2:1), between 15 s ($U = 2.0$ km/s) and 40 s ($U = 4.0$ km/s), corresponding to $k_{min} = 0.039$ km^{-1} and $k_{max} = 0.113$ km^{-1}. Measurements are taken at $T = 20$ s ($k_0 = 0.080$ km^{-1}; $U_0 = 3.6$ km/s; $\alpha = 0.08$ km/s^2). Using these values, we computed numerically the relevant values of ξ when the epicentral distance Δ varies from 1° to 170° (D varies from 111 to 18,903 km). Figure 1 shows that as long as $\Delta \geq 10°$, the two complex erf functions in (11) can be replaced by their asymptotic value for infinite x (one), yielding

$$a = \frac{1}{\pi}\frac{A(\omega_0)\, U_0\, \omega_0}{\sqrt{\alpha D}} = 2\frac{A(\omega_0)\, U_0}{T_0\sqrt{\alpha D}} \quad (12)$$

where $T_0 = 20$ s is the period at which the measurement is taken, the error introduced between (11) and (12) being within ±0.05 units of magnitude. This result is a direct illustration of the method of phase-stationary integrals [*Erdélyi*, 1956]. However, by detailing its derivation, we can discuss the validity of all successive approximations.

First, we still must justify the approximation in (3). The phase-stationary method will work if the rate at which the spectral amplitude $A(\omega)$ varies with frequency is slower than the rate of change of the phase under the integral sign in (6). While A is a somewhat complex function, not only of ω but also of the focal geometry and depth of the source, an estimate of its behavior with frequency can be obtained by averaging its values over a wide range of these parameters [*Okal and Talandier*, 1987; this issue]. Figure 2 is a close-up of Figure 3 of *Okal and Talandier* [this issue], showing the variation with period of the logarithmic average excitability for a

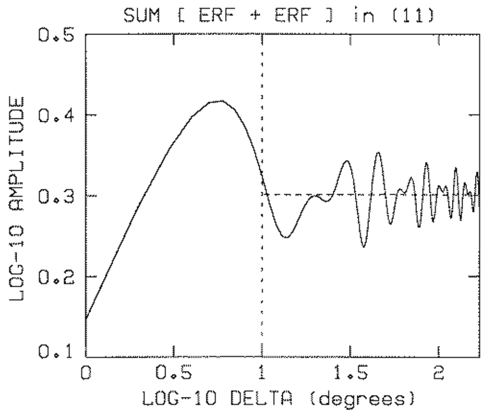

Fig. 1. Behavior of the "erf" functions in (11) in the case of 20-second Rayleigh waves. The function plotted is the logarithm (base 10) of the modulus of the sum of the two complex erf functions in (11), as a function of the logarithm of epicentral distance in degrees. The figure shows that to the right of the dashed line ($\Delta \geq 10°$), the erf functions can be approximated by their asymptotic value, 1, with an error of at most 0.05 units of magnitude.

number of depths ranging from 10 to 75 km. This quantity can be defined as the logarithm of an average (taken over many focal geometries) of $A(\omega)$ for a unit moment of 10^{27} dyn-cm. The thick line is the spline used by *Okal and Talandier* [this issue] to describe $A(\omega)$; except for the two deepest sources (60 and 75 km), it gives an adequate estimate of the excitability. The slope of this curve shows that in the vicinity of $T = 20$ s, A grows approximately like $\omega^{2.5}$ (slower at longer periods, and if anelastic attenuation is taken into account). If we define a characteristic time $\tau_A = [A(\omega)]^{-1}\, dA(\omega)/d\omega$, this suggests that τ_A varies from 6 s at $T = 15$ s to 11 s at $T = 35$ s. This quantity must be compared to $1/\delta\omega$, where $\delta\omega$ is the change of frequency necessary to flip the phase of the exponential in (6). It is easy to show that $\delta\omega = U_0\omega_0/\sqrt{\alpha D}$, so that in the end the approximation (3) requires

$$D > \frac{4\pi^2}{\alpha}\, U_0^2 \left[\frac{\tau_A}{T_0}\right]^2 \tag{13}$$

At $T = 20$ s, this is equivalent to $D > 1023$ km, or in round numbers, $\Delta > 10°$.

Taking into account the effect of anelastic attenuation would reduce the dependence of $A(\omega)$ on ω by subtracting from τ_A a term $D/2UQ$, which is about 0.5 s at $D = 1000$ km, and in practice does not affect the constraint (13)

An additional problem could be that of the source phase ϕ_0 in (3), which in principle also varies with frequency if the mechanism is not "pure", i.e., involving either only horizontal motion or only vertical slip on a purely vertical fault plane. This is due to the variation with frequency of the three independent excitation coefficients. The latter can be expressed in a variety of ways, such as K_0, K_1, K_2 in the formalism of normal modes [*Kanamori and Cipar*, 1974] or K_1^*, K_2^*, K_3^* in the formalism of surface waves [*Okal*, 1982]. The exact expression of their variation with frequency is complex, but the following order-of-magnitude argument can be made in the case of the Rayleigh wave of a homogeneous half-space: The variation with frequency of the coefficients K is controlled by the decaying factors of the compressional and shear potentials, $\exp[-\eta\omega z/C]$, where z is the focal depth, C the phase velocity, and $\eta = 0.39$ for the shear potential; 0.85 for the compressional one. Therefore the characteristic time for the variation of K_i^*, and hence of ϕ_0, is

$$\tau_\phi = \left| \frac{1}{K_i}\frac{dK_i}{d\omega} \right| = \eta\, \frac{z}{C} \tag{14}$$

For a typical earthquake, $z = 30$ km, $C = 4$ km/s, leading to τ_ϕ on the order of 6 s or less. This is much larger than $1/\delta\omega$ used to discuss (13), and the variation of ϕ_0 can be neglected.

In practice, the measurement of time domain amplitudes is usually carried out directly on a seismogram, and the ground motion is retrieved by simply dividing the measured amplitude a by the instrument response at the appropriate frequency $R(\omega_0)$. Thus we must also discuss the influence of the variation of $R(\omega)$ with frequency on the validity of applying phase-stationary asymptotics. Most M_s measurements are taken on so-called "long-period" instruments, such as the WWSSN LP, peaked at or close to 20 s. R is actually stationary at this period, and its average falloff in the ranges 10–30 s is approximately 3 dB per octave, leading to a characteristic time $\tau_R \cong 1.5$ s. This contribution is again negligible when compared to τ_A. As for the phase ϕ_R of the instrument response, its variation $d\phi_R/d\omega$ is approximately $\pi/2\omega_0$ for a critically damped instrument; this figure, on the order of 5 s, must be compared with the much faster variation of the phase term due to propagation, $k(\omega)D$. In brief, the influence of instrument response on the validity of the phase stationary asymptotics is negligible for a long-period system.

Thus we conclude that the range of validity of the approximation in (3) is $\Delta > 10°$; its being identical to the range of validity of the asymptotic expression of the erf functions in (11) is fortuitous, since it involves the properties of the excitation coefficients controlling $A(\omega)$ which are not involved in (11).

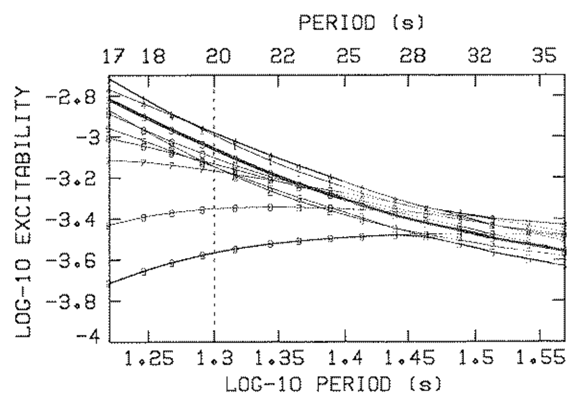

Fig. 2. Logarithmic average excitability, as defined by *Okal and Talandier* [this issue], plotted for Rayleigh waves as a function of period. The various symbols (from 0 to 9) refer to 10 source depths, from 10 to 75 km. The thicker trace corresponds to $h = 20$ km, which is retained for the computation of C_S in (15).

Equation (12) relates the time domain amplitude $a(t)$ to the spectral amplitude $A(\omega)$ of the dispersed wave. The influence of the parameters α and D is straightforward: they control the amount to which the wave is "spread out" over the group time t. There remains to relate $A(\omega)$ to the physical size of the source, i.e., to the seismic moment M_0. In *Okal and Talandier* [1987, this issue], we have introduced a mantle magnitude M_m based on the measurement of the spectral amplitude of very long period Rayleigh waves. We showed in particular that the relationship between the moment M_0 and the spectral amplitude $A(\omega)$ could be written in the form

$$\log_{10} M_0 = M_m + 20$$
$$= \log_{10} A(\omega) + C_S + C_D + 19.10 \qquad (15)$$

In this formula, M_0 is in dyne-cm and A in μm-s. Equation (15) remains valid at any period, with adequate values of the source and distance corrections C_S and C_D. Thus the above results suggest the following relation between the amplitude of the 20-second Rayleigh wave in the time domain, a, and M_0:

$$\log_{10} M_0 = \log_{10} (a\ T) + C_S + C_D + 0.5 \log_{10} \Delta + \log_{10} \left[\frac{\sqrt{\alpha\ l_{deg}}}{2U}\right] + 19.10 \qquad (16)$$

where Δ is now in degrees and $l_{deg} = 111.2$ km is the length of 1°.

The source correction C_S expresses the variation of the excitation of Rayleigh waves with frequency. This term is a priori dependent on focal mechanism and depth. The philosophy of a magnitude measurement is that these parameters are left unknown and not corrected for. We refer to the above two papers for a detailed method of averaging this correction over a large number of focal geometries and for a discussion of the effect of an unknown hypocentral depth. Figure 2 justifies the use at 20 s of the cubic spline proposed by *Okal and Talandier* [this issue] and shown as the thick line, for depths ranging between 10 and 50 km. The corresponding value at 20 s is $C_S = 3.06$. With M_s given by the Prague formula the expected relationship between M_0 and M_s results from the two equations:

$$\log_{10} M_0 = \log_{10} (aT) + [C_D + 0.5 \log_{10}\Delta] + 21.78 \qquad (17a)$$

$$M_s = \log_{10} (aT) - 2 \log_{10} T + 1.66 \log_{10} \Delta + 3.3 \qquad (17b)$$

In (17*b*), we have artificially introduced the product (aT) in order to facilitate a direct comparison between M_0 and M_s.

4. DISCUSSION OF THE PRAGUE FORMULA

Justification of 1.66 $\log_{10} \Delta$

The distance correction C_D valid in the spectral domain is composed of a term $0.5 \log_{10} \sin\Delta$ due to geometrical spreading on the spherical Earth and of a term $[(\log_{10} e)\ \omega D/2UQ]$, which corrects for anelastic attenuation during propagation. In the time domain the additional term $0.5 \log_{10} \Delta$ expresses the fact that as distance increases, the energy in any small frequency band around ω_0 is spread out over increasingly large group time intervals. We define the full time domain distance correction as $C_D^{TD} = C_D + 0.5 \log_{10} \Delta$. This general form of the effect of distance on the time-domain amplitude of a dispersed signal was given by *Ewing et al.* [1957]. They did not, however, discuss in detail the range of validity of the approximations involved. Furthermore, their book predates the development of the modern concept of seismic moment and to a large extent the present framework of the theory of the excitation of surface waves; as such, they did not directly relate a to the earthquake size, described nowadays by M_0. We seek to justify that C_D^{TD} can be modeled as $1.66 \log_{10} \Delta + b$.

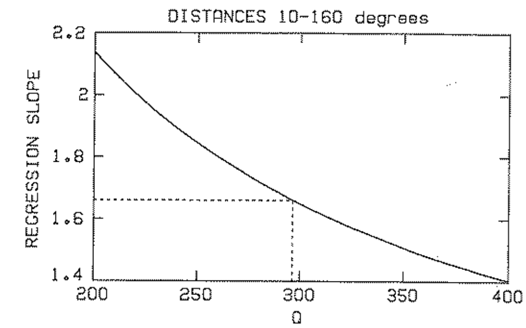

Fig. 3. Influence of Rayleigh Q on the behavior with distance of the term C_D^{TD}. For each value of Q this term is regressed as a function of $\log_{10} \Delta$. The corresponding slope is plotted. A slope of 1.66 is obtained for $Q = 297$.

At distances greater than 20°, *Nuttli* [1973] showed that the form of distance decay used empirically for 20-s amplitudes ($-1.66 \log_{10} \Delta$) would approach the theoretical expression proposed by *Ewing et al.* [1957], assuming a distance attenuation coefficient $\gamma = 0.0015$ deg^{-1}. This would correspond to a relatively high value of Q = 325–390, depending on the exact dispersion curve used. In addition, he was working under the assumption of an Airy phase, which is probably unwarranted at 20 s. In the present study we assume that the magnitude measurement is taken on a strongly dispersed branch, away from an Airy phase, and seek whether an adequate value of Q can reconcile the two distance corrections in (17*a*) and (17*b*).

Vaněk et al. [1962] indicate that the Prague formula can be used between 2° and 160°. However, we have shown that the approximation of the erf functions in (11) is expected to deteriorate significantly for $\Delta \leq 10°$. Similarly, the separation of the amplitude out of the integral in (3) will not be valid below 10°. We thus investigate the behavior of the theoretical correction C_D^{TD} in the range Δ = 10°–160°, with Q varying between 200 and 400. For each value of Q, we regressed C_D^{TD} against $\log_{10} \Delta$; Figure 3 shows that the resulting slope depends significantly on Q. A slope of 1.66 is achieved for $Q = 297$. The corresponding zero-intercept is $b = -1.623$. Figure 4 shows the resulting fit between the two corrections.

Because of the extreme heterogeneity of crustal structures, it is difficult, if at all meaningful, to try to define an average Q for Rayleigh waves at 20 s. Using the work

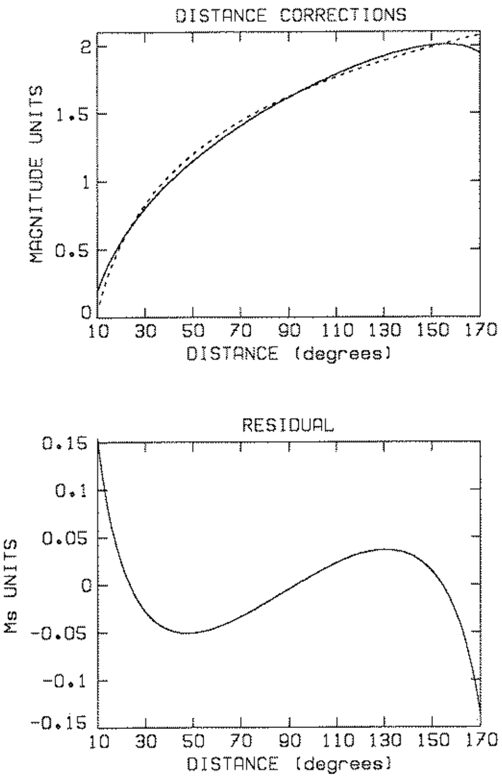

Fig. 4. (Top) Theoretical distance correction C_D^{TD} as a function of distance (solid curve), compared to its approximation through the Prague formula (dashed curve). (Bottom) Residual $C_D^{TD} - 1.66 \log_{10} \Delta + 1.623$ of the two distance corrections, plotted as a function of distance. Note that the fit deteriorates significantly below 20° and beyond 150°.

of *Canas and Mitchell* [1978] and the regionalization proposed by *Okal and Talandier* [this issue], the average oceanic Q is found to be 261. *Hwang and Mitchell* [1987] suggest values as high as 360–400 for the South American shield and values around 120–130 in tectonically active continental areas. A global average using these values would yield $Q = 240$. However, the geographical sampling of the Earth's surface by Rayleigh wave paths is far from homogeneous. Similarly, the regression in Figure 2 weights all distances equally, while M_s measurements are rarely taken beyond 140° and may be made at periods significantly shorter than 20 s at short distances [*Vaněk et al.*, 1962]. We conclude that the value $Q = 297$ explaining theoretically the coefficient 1.66 is not improbable as an average of the known attenuation properties of Rayleigh waves between realistic epicenter-station pairs. In addition, we note that in early works, a higher value was often found for the distance correction coefficient e.g., 2.0 [*Gutenberg and Richter*, 1942]. This can be understood if the values of Q involved were lower, as indeed would be expected at Pasadena, where shields paths are few and the influence of the highly attenuating Western U.S. structures is important.

Assuming the optimal Q value of 297, we have shown that C_D^{TD} can be modeled as $1.66 \log_{10} \Delta - 1.623$. Figure 4 shows that the residual between this expression and the exact value of the distance correction never exceeds ± 0.05 units of magnitudes at distances between 20° and 150°. As expected, the fit starts to deteriorate for $\Delta \geq 150°$ due to the focusing of energy at the antipodes. This behavior, correctly described by C_D^{TD} but ignored in (1), results in overcorrecting of the distance effect by the Prague formula.

On the other hand, at distances shorter than 20°, the residual on Figure 4 becomes positive, the Prague formula underestimating the distance correction. This is because the term $1.66 \log_{10} \Delta$ is designed among other things to compensate for the effect of attenuation; at short distances the modeling of the attenuation by a logarithmic rather than a linear function is significantly deficient. In addition, at short distances, the effect of the dispersion can no longer be described through the phase stationary asymptotics, since the approximation between (11) and (12) is not expected to hold. In the vicinity of an Airy phase it is in principle possible [e.g., *Nuttli*, 1973] to replace the $\Delta^{-1/2}$ dependence in (12) with the $\Delta^{-1/3}$ expected from a higher-order expansion of (10) [*Jeffreys*, 1925; *Ewing et al.*, 1957]. On the other hand, *Gutenberg* [1932] and *Gutenberg and Richter* [1936] have proposed that amplitudes decay as $\Delta^{-1/4}$ (and $\Delta^{-1/6}$ at short distances), but the origin of the proposed exponents -1/4 and -1/6 remains obscure. Furthermore, neither of these authors considered the variation of $A(\omega)$, which, as discussed above, can invalidate (3) in the first place. Finally, it is clear that propagation along very short paths, in the range $\Delta \leq 10°$, cannot achieve the geographical averaging of attenuation properties inherent in the use of any worldwide formula. This was noted by *Alewine* [1972] who proposed to correct the expression of the magnitude scale at short distances, using regionalized Q models.

We conclude that despite several problems at shorter distances, the use of the correction $1.66 \log_{10} \Delta$ can be justified in the usual range of M_s measurements.

M_s–M_0 Relationship

After combining (17) with the results of the previous section, we find that the following relation is expected between M_s and M_0:

$$\log_{10} M_0 = M_s + 19.46 \tag{18}$$

Obviously, this expression will be valid only when the source can be described as a space-time δ function, i.e., for the sufficiently small earthquakes which do not involve source finiteness effects leading to the distortion and eventual saturation of M_s. *Geller* [1976] suggests that this corresponds to $\log_{10} M_0 \leq 25.6$; *Ekström and Dziewonski* [1988] give the more stringent condition $\log_{10} M_0 \leq 24.5$.

Our theoretical relationship (18) compares very favorably with its empirical counterparts published in the literature: on the basis of a large data set of more than 2000 events, *Ekström and Dziewonski* [1988] propose a constant of 19.24. *Geller* [1976] proposes a significantly lower value (18.87); similarly, *Purcaru and Berckhemer* [1978] proposed a constant of 18.8 on the basis of about 25 events with $M_s \leq 5.5$. The nature of the data sets used to obtain these values is not given. J. Talandier (personal communication, 1988) obtained an average of 19.59 ± 0.23 for a data set of 323 events measured at Papeete, Tahiti; the worldwide average for these events

TABLE 1. Variation With Period T of the Principal Parameters in (16)

T, s	C_S	Q	C, km/s	U, km/s	C_D † ($\Delta = 60°$)	α, km/s^2	$\log_{10} \frac{\sqrt{\alpha}}{U}$	C_M	$2 \log_{10} T$	Δ_{min}, deg
15	2.685	350	3.71	2.00	0.867	0.5	-0.451	3.101	2.352	
17	2.865	325	3.80	3.00	0.549	0.3	-0.739	2.675	2.461	15
19	3.006	310	3.88	3.30	0.468	0.15	-0.930	2.544	2.558	9
20	3.065	297	3.93	3.60	0.426	0.08	-1.105	2.386	2.602	10
21	3.120	285	3.95	3.65	0.417	0.065	-1.156	2.381	2.644	9
23	3.212	270	3.97	3.70	0.396	0.05	-1.219	2.389	2.723	14
25	3.288	250	3.99	3.75	0.388	0.03	-1.336	2.340	2.796	30
30	3.430	225	4.01	3.85	0.350	0.02	-1.435	2.345	2.954	500
35	3.524	200	4.02	3.95	0.329	0.015	-1.509	2.344	3.088	

†Only the second, frequency-dependent term in C_D is compiled.

was 19.25, practically equal to Ekström and Dziewonski's, the difference with Papeete (0.34 units) pointing out to the significant influence of regional structures.

Furthermore, as mentioned above, for such small earthquakes, few records will be obtained at large distances, and M_s will most probably be measured at distances not exceeding 110°. For this range of distances, C_D^{TD} is actually overestimated by the Prague distance correction by 0.02–0.04 units (Figure 4). This acts to slightly narrow down the difference between our constant and *Ekström and Dziewonski*'s [1988] empirical average.

Finally, the history of the exact origin of the term 3.3 in the Prague formula is somewhat confused. *Vaněk et al.* [1962] indicate that this constant must be regarded as an average of many individual and empirical compilations. G. Purcaru (personal communication, 1988) has indicated to us that the scatter involved in this average may reach 0.4–1.3 unit of magnitude, depending on such factors as the number of stations used in the averaging. Given this estimate of the stability of the magnitude scale M_s, the fact that we derived theoretically an M_s–M_0 relationship upheld experimentally within 0.2 units of magnitude and falling between a world wide average and the average for a large data set at an oceanic station must be regarded as satisfactory.

The Question of (a/T) *Versus* $(a\ T)$

One of the most important results of this analysis is that we predict that the seismic moment is related to the time domain amplitude through the product (aT) of amplitude and period, rather than through their ratio, commonly used in many magnitude scales, including in the Prague formula. Hence the term $-2 \log_{10} T$ in (17*b*). The use of (a/T) stems from early attempts [e.g., *Gutenberg and Richter*, 1942] to measure the total elastic energy released by the earthquake source, the energy of a perfect harmonic oscillator being, of course, proportional to a^2/T^2. However, this is true only for a monochromatic oscillator. The seismogram recorded on an instrument with a narrow-band response may look monochromatic, but modern source theory has taught us that the spectrum of the seismic source itself is basically flat below the corner frequency and that the total energy released is proportional to M_0 and thus to the supposedly constant value of $A(\omega)$ below the corner frequency. Why then does the time-honored practice of using a/T in (1) work, if at all?

It is important to realize that most M_s measurements are taken at 20 s, or very near that reference period. Then the term $-2 \log_{10} T$ is practically constant and can be absorbed into the final constant, as is often done. For example, it is customary, especially in Western countries, to restrict the measurement of M_s to a narrow range of periods, in practice 17–23 s [*Purcaru and Berckhemer*, 1982]. However, observatories in the Soviet Union and Eastern Europe occasionally report M_s values taken at more scattered periods, as discussed briefly in the original Prague formula paper [*Vaněk et al.*, 1962]. In view of this practice, we will discuss the theoretical bias which can be expected if the period is allowed to change substantially around 20 s. This discussion will also be important for the extension of the magnitude concept to mantle waves in the following section.

When the reference period T is changed, the constants in M_s given by (1) are unchanged. However, many parameters in (16) can be expected to vary significantly with T. Table 1 lists their values at selected periods between 15 and 35 s, based on *Oliver*'s [1962] average dispersion models.

The correction C_S, easily computed from Equation (10) of *Okal and Talandier* [this issue], increases by 0.84 units between 15 and 35 s. In C_D, only the second term, correcting for anelastic attenuation, is period-dependent. It is somewhat more difficult to evaluate, since at each period, we need in principle a new Rayleigh Q. Also, its variation with period is itself distance dependent. In Table 1 we use a typical distance of 60° and assume a world average Rayleigh Q decreasing from 350 at 15 s to 200 at 35 s. The decrease of C_D is 0.54 units from 15 to 35 s.

The strongest variations are those of U (from 2 to 3.95 km/s) and above all α (from 0.5 km/s^2 at 15 s to 0.015 km/s^2 at 35 s). Table 1 lists as

$$C_M = C_S + (\log_{10} e)\, \frac{\omega D}{2UQ} + \log_{10} \frac{\sqrt{\alpha}}{U} \qquad (19)$$

the sum of all period-dependent terms in (16). This quan-

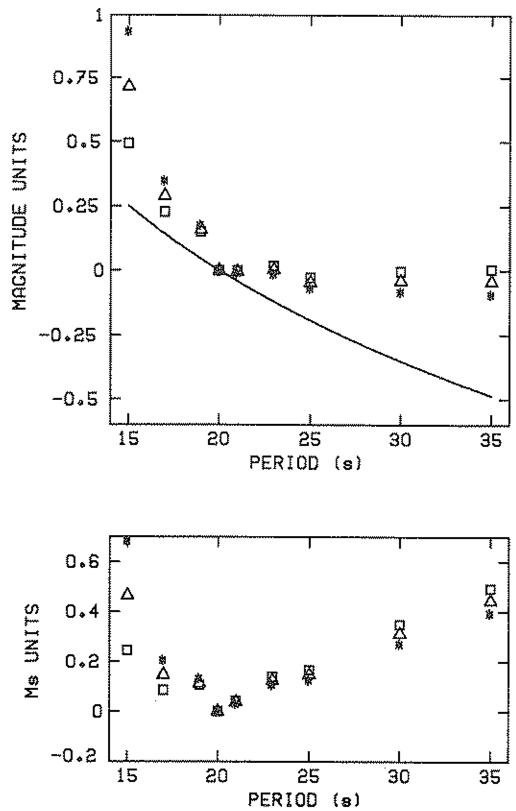

Fig. 5. (Top) Variation of the term C_M defined in (19) with the period of measurement. The quantities plotted are the residuals of C_M with respect to its value at $T_0 = 20$ s, for $\Delta = 30°$ (open squares), 60° (open triangles; see values listed in Table 1), and 90° (asterisks). These values are compared to the correction $[-2 \log_{10} (T/T_0)]$ introduced by the use of a/T in the Prague formula (solid curve). (Bottom) Systematic bias $C_M - C_M(T_0) + 2 \log_{10} (T/T_0)$ introduced through the use of Prague formula; symbols as in the top frame.

tity drops about 0.7 units of magnitude between 15 and 20 s but remains fairly constant at longer periods.

The computation of M_s ignores the variation of this term but replaces it arbitrarily with $-2 \log_{10} T$, which is plotted as the solid curve on Figure 5. This in effect reduces the systematic error to at most 0.46 units of magnitude. Within the range 17–23 s adopted by most Western observatories, the variation remains on the order of 0.15 units. Figure 5 also shows that these results are not significantly affected by the use of a different distance.

In conclusion, the two most period-dependent terms are the correction for anelastic attenuation (which is also distance-dependent) and the parameter α, characterizing the dispersion of the wave. It is clear that α should have an important effect on the amplitude of the wave, since in effect it controls the spreading with group time of the energy contained in a narrow frequency band around ω_0. Both of these parameters are completely ignored by the Prague formula. It so happens that the $1/T^2$ "correction" compensates partially for their absence, reducing the systematic error in the 17–25 s period band to within the level of ± 0.2 units, which is generally accepted as characteristic of the precision of magnitude scales.

In addition to the deterioration of the $1/T^2$ correction at larger periods, we want to emphasize, that because of substantially reduced values of α resulting from the presence of an Airy phase at 40 s, the use of the Prague formula at longer periods and short distances quickly becomes improper. The last column in Table 1 shows the minimum distance Δ_{min} at which it is legitimate to replace the erf functions in (11) by 1; we obtained these values by requiring that the real argument of the function ξ remain larger than 1.7. Clearly, this approximation breaks down very fast as the period is increased beyond 25 s. By comparison, the domain of invalidity of the approximation (3) is found to increase only much slower, from about 0.5° at 15 s to 26° at 30 s.

The fact that a/T does not provide a satisfactory, stable estimate of the size of the earthquake source was pointed out at much shorter periods by *Herrmann and Nuttli* [1982], who proposed to replace *Nuttli*'s [1973] m_{bLg} regional magnitude featuring a term $\log_{10} a/T$, with an $m_{Lg}(f)$, featuring only $\log_{10} a$. As discussed more in detail in the next section, such a behavior would be expected when the arguments of the erf functions in (11) become very small. This could be the case at the short periods characteristic of Lg; however since Lg actually consists of the superposition of many surface modes [*Knopoff et al.*, 1973], it is difficult to define constants such as α in (11), and a direct comparison of the results is impossible.

Finally, we want to stress that according to the Prague convention, M_s is supposedly measured on the horizontal component; as such it may be taken on a Love wave; in the present study we have assumed that a is taken on the vertical component of the Rayleigh wave. This follows the practice, common in recent years for the National Earthquake Information Service and other agencies, to report measurements taken on the vertical component ("$M_s Z$"). This eliminates the need to rotate the horizontal components; in addition, the vertical displacement of a Rayleigh wave is more robust than the horizontal one with respect to changes in crustal structure; finally, as discussed by *Geller and Kanamori* [1977], the contamination with Love waves and overtones may offset the bias of 0.15 units expected from the surface ellipticity of a Rayleigh wave.

5. EXTENSION TO MANTLE WAVES: THE TIME-DOMAIN MAGNITUDE M_m^{TD}

The purpose of this section is to discuss the extension of a formula of the type (12) to the inversely dispersed branch of mantle Rayleigh waves between 50 and 250 s. We are motivated in this study by (15), which defines a spectral domain mantle magnitude, M_m [*Okal and Talandier*, this issue]. We seek to extend this concept to a measurement taken in the time domain, and specifically to justify theoretically the formula used by *Okal and Talandier* [this issue; Equation (16)]:

$$M_m^{TD} = \log_{10} (aT) + C_D + C_S - 1.20 \qquad (20)$$

Equation (11) remains valid for mantle waves under the condition of sufficiently strong dispersion. In practice, the bounds of the dispersed branch will now be $\omega_{min} = 2\pi/250$ and $\omega_{max} = 2\pi/50$ s^{-1}, respectively.

The applicability of (11) requires that the measurement not be taken in the vicinity of an Airy phase. In the case of mantle waves this means at periods of between 60 and 230 s. Measurements taken around 50 or 250 s could be invalid. In addition, the minimum distance at which the spectral density $A(\omega)$ can be taken out of (3) is still given by (12), with obviously different values of the constants. Our results on Rayleigh wave excitation [*Okal and Talandier*, this issue; Equation (10)] suggest that its contribution to τ_A is now on the order of 0.5 s. Along the inverse branch, α is typically on the order of 10^{-3} km/s^2, resulting in $D > 200$ km. This condition is not stringent, since at such close distances, the Rayleigh wave is not properly developed in the first place. In addition, anelastic attenuation also contributes to the variation of A with frequency, actually resulting in an upper bound on D, found to be on the order of 100,000 km. In short, (12) holds for mantle waves at all distances typical of teleseismic Rayleigh waves.

The next step in our study of M_s was to replace the erf functions in (11) with their asymptotic values (one) for infinite argument. However, in the present case, this approximation is no longer possible, the main reason being that α takes considerably smaller values: from 2.5×10^{-4} km/s^2 at 60 s, to a maximum of 2.1×10^{-3} km/s^2 at 140 s (as opposed to 0.08 km/s^2 for 20-s Rayleigh waves); in addition, the ratios of k to ω, i.e., the phase slownesses $1/C$, are also smaller. Under these conditions we study systematically the variation of a, as given by (11) for unit $A(\omega)$, with period and distance. For each value of the period T between 60 and 230 s, we use specific values of k, U, and α. We let the distance vary between 20° and 120°, which is representative of the data set at Papeete, Tahiti, used by *Okal and Talandier* [this issue]. Extension to other distances is discussed later. Results are shown on Figure 6*a*, on which the various lines and symbols correspond to different values of T.

For most values of period and distance the moduli of the complex arguments of the erf functions in (11) are of order 1, leading to oscillations in the resulting value of a. While the data set on Figure 6*a* is best fit as a function of $\log_{10} T$ with a slope of −1.4, Figure 6*b* shows that modeling of a/A as $2/T$ yields residuals no larger than ±0.22 magnitude units, for periods no greater than 200 s. On the other hand, dependence on distance is very weak, with a best fit slope of only −0.2 . Physically, this means that in this period and distance ranges, dispersion is strong enough to build up a mantle Rayleigh wave in which each successive oscillation, or arch, has an apparent period significantly different from the previous one, a situation familiar to observational seismologists and which was the basis of *Okal and Talandier*'s [1987] modeling of the relationship between time domain and spectral domain amplitudes (see their Figure 2):

$$a/A = 2/T \tag{21}$$

For low values of T, and especially at the shorter distances, the ratio a/A is found practically constant and equal to 0.03 Hz. Mathematically, in this regime the two functions erf(z) in (11) can be replaced by their equivalent for very small arguments, $(2/\sqrt{\pi})\, z$, leading to

$$a \ / \ A(\omega) = \frac{1}{\pi}\ U\ (k_{\max} - k_{\min}) = 0.0315 \text{ Hz} \tag{22}$$

Physically, this corresponds to circumstances (both α and D small) under which dispersion is too weak, with practically all energy in the wave arriving at the same time, and the signal comprising only one seismic phase, as shown on Figure 15 of *Okal and Talandier* [this issue]. If condition (12) is fulfilled, i.e., beyond a distance of a few degrees, time domain measurements could still be taken in principle, but using the constant ratio of 0.0315 Hz between a and A. As mentioned above, this situation may correspond to the case of Lg for which *Herrmann and Nuttli* [1982] have proposed using the amplitude a instead of a/T in the computation of a magnitude.

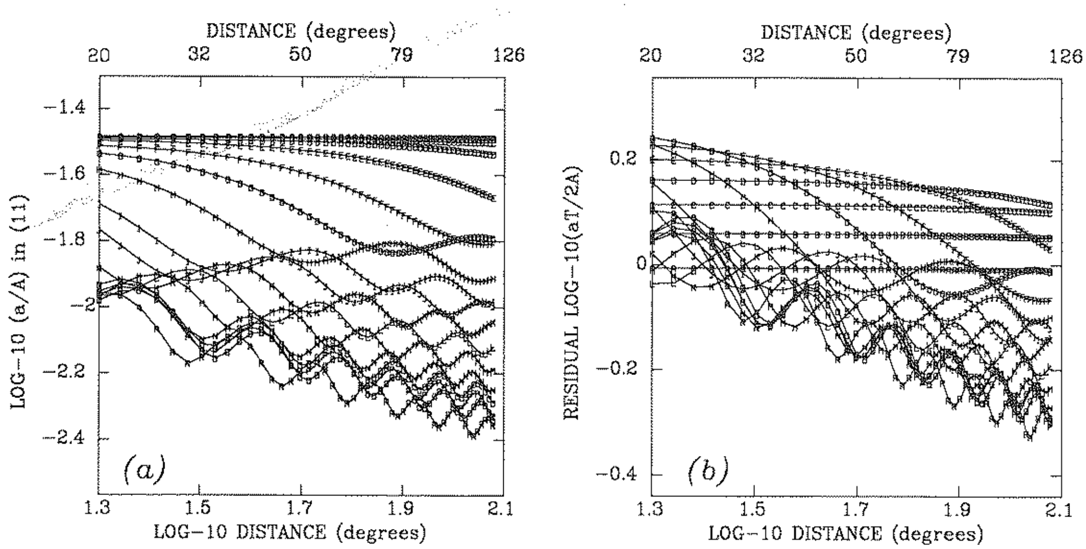

Fig. 6. Behavior of the ratio a/A, as defined in (11), as a function of distance, and for selected periods in the range 60–230 s. *(a)* $\log_{10}(a/A)$ plotted as a function of $\log_{10} \Delta$. Each trace and symbol correspond to an individual period, from 60 s (A) to 230 s (R), in increments of 10 s. *(b)* Residual resulting from modeling of a/A as $2/T$, as a function of distance.

On the other hand, if distance is increased greatly, for example, if we consider multiple passages of the Rayleigh wave, dispersion becomes too strong, i.e., group times at the station for significantly different periods are separated by more than the typical period of the mantle Rayleigh wave. The following simple calculation based on *Okal and Talandier*'s [1987] model can be used to estimate a minimum distance for this regime. Assuming we take a measurement at a period $T_0 = 2\pi/\omega_0$, (21) will be valid only if the neighboring oscillation in the record has a frequency ω such that $\mathrm{sinc}[\frac{1}{2}(\omega_0 - \omega)T_0] \ll 1$, or $(\omega_0 - \omega)T_0 > 2\pi$. The difference in frequency between two successive arches, due to dispersion, is simply $\omega_0 - \omega = 2\pi\, U^2/T\, D\, \alpha$, leading to $D < U^2/\alpha$, typically on the order of 15,000 km, or 150°. Beyond this distance, the arches in *Okal and Talandier*'s [1987] simple model would have apparent periods which are too close to each other, and (21) no longer applies. Rather, we reach the regime in which the erf functions can be replaced by 1, since their arguments are themselves large compared to 1; an additional correction to (20) of the form $0.5 \log_{10} \Delta$ is anticipated.

These results are summarized in Figure 7, in which we have extended our distances to 900°, corresponding to multiple passages up to R_5 (at greater distances, the effect of anelastic attenuation prohibits using phase-stationary asymptotics). Realistically, and for such large distances, we consider only periods greater than 100 s. Then, a formula of the form

$$\log_{10} a = \log_{10} X(\omega) - \log_{10} \frac{2}{T} - 0.5 \log_{10} \frac{\Delta}{70} \quad (23)$$

where Δ is epicentral distance in degrees, leads to theoretical residuals no larger than ±0.2 magnitude units. On Figure 8, we check this result against real data by plotting time domain magnitudes obtained using (20) from GEOSCOPE records on the Mexican earthquake of April 30, 1986. The methodology of these measurements is similar to that given by *Okal and Talandier* [this issue]. In particular, and in order to eliminate the influence of focal mechanism, we plot residual differences between a multiple passage and the first passage R_1 at the same station. Magnitudes are clearly underestimated at the very large distances corresponding to R_3 and R_4. A regression of this data against $\log_{10} \Delta$ yields a slope of −0.45, in excellent agreement with (23). This trend disappears if the distance correction in (23) is used. The change in behavior of the distance dependence of the time domain amplitude of Rayleigh waves beyond 120–160° was also addressed by *Brune and Engen* [1969], at the constant period of 100 s; however, these authors did not provide a full derivation of the empirical relationship they proposed between a and A.

When distance is increased substantially (i.e., starting beyond 120°), the effect of dispersion becomes too strong for (22) to be used, and the distance correction in (23) must be added, which results in replacing (20) with

$$M_m^{TD} = \log_{10} [a\, T] + C_S + C_D + 0.5 \log_{10} \Delta - 2.12 \quad (24)$$

With this precaution, it is legitimate to use time domain mantle magnitudes for multiple passages of Rayleigh waves, up to R_5.

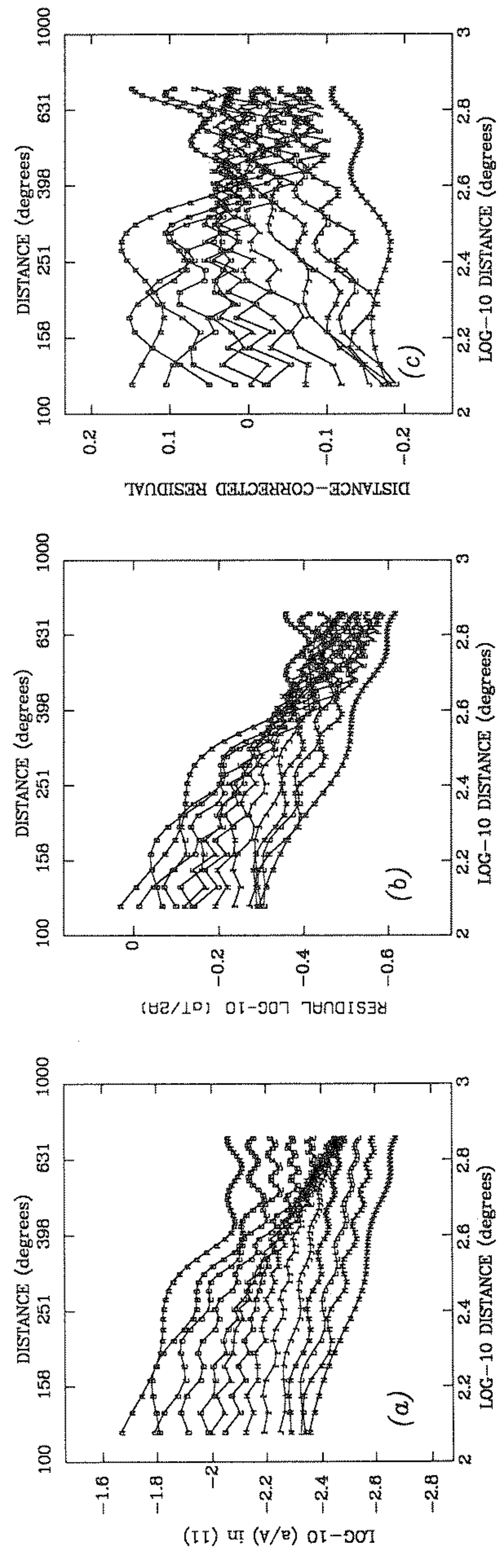

Fig. 7. (*a*) and (*b*) Same as Figure 6 in the distance range 100–720°, and for periods ranging from 100 s (A) to 230 s (N). (*c*) Same as Figure 7(*b*), but with the additional correction $0.5 \log_{10} (\Delta/70)$.

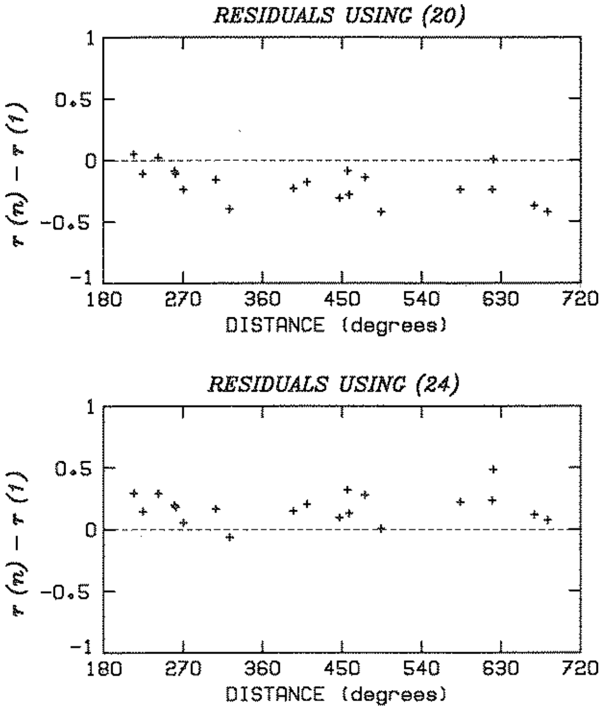

Fig. 8. Time domain residuals of M_m^{TD} for 20 GEOSCOPE records of multiple Rayleigh waves of the Mexican earthquake of April 30, 1986. (Top) Raw residuals, obtained by applying (20). (Bottom) Distance-corrected residuals obtained by computing M_m^{TD} through (24).

6. CONCLUSIONS

The principal conclusions of this study can be summarized as follows:

1. The combination of the theory of propagation and excitation of Rayleigh waves with phase-stationary asymptotics leads to an expression of the type (16) between the time domain amplitude of a strongly dispersed wave and the seismic moment of the parent earthquake; this expression ceases to be valid if the real argument of the erf functions in (11) become less than about 1.7, corresponding to $\Delta \leq 10°$ at 20 s but increasing rapidly to $\Delta \leq 30°$ at 25 s.

2. The total distance correction in (16) can be adequately modeled by a linear function of $\log_{10}\Delta$ over the usual range of magnitude measurements (from 10° to 160°). A slope of 1.66 is found if the Rayleigh Q takes the average value 297 at 20 s. This figure is somewhat lower than proposed by *Nuttli* [1973], reflecting a broader range of distances, and his use of an Airy phase formalism at 20 s.

3. With this value of Q, the relationship (18) is predicted for small earthquakes between M_s and M_0. The value 19.46 is in good agreement with the compilation by *Ekström and Dziewonski* [1988]. To our knowledge, this constitutes the first theoretical justification of all constants in a magnitude-moment relationship.

4. The theory predicts that M_0 should be related to the product (aT) of amplitude and period, not to their ratio. That magnitude measurements using (a/T) can occasionally be taken at periods other than the reference period reflects a partial, and ad hoc, compensation for a large number of period-dependent terms ignored in the Prague formula. The use of (a/T) cannot be justified theoretically for a strongly dispersed wave. Furthermore, outside the narrow interval 17–23 s, it is expected to lead to significant bias.

5. The same theoretical analysis used to justify the Prague formula shows that it is legitimate to use an equation such as (21) to relate time domain and spectral amplitudes for periods between 60 and 230 s, at distances between 20° and 120°. This justifies theoretically the use of time domain measurements for the purpose of obtaining a mantle magnitude, as given by (20), and reported by *Okal and Talandier*'s [this issue] Table 7 and Figure 16. At shorter distances a better formula would be (22), but the use of (20) brings errors no larger than ±0.22 units.

6. At larger distances, dispersion becomes too strong for (21) to hold; an additional distance correction is required, and (24) must be used. This correction must be effected beyond $\Delta = 150°$; then time domain measurements on multiple passages are valid in principle up to R_5. At even greater distances it is anticipated that the effect of anelastic attenuation on surface wave amplitude would prohibit the use of phase-stationary asymptotics.

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