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STACKING INVESTIGATIONS OF HIGHER-ORDER MANTLE RAYLEIGH WAVES

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Abstract. We present the initial results of an ongoing study of higher-order mantle overtones of Rayleigh waves. The main problem is to separate overtone branches ($_3R$,$_4R$,$_5R$), traveling with identical group velocities at similar frequencies. Global stacking is used, both for traveling waves and normal modes, with dispersion results in generally excellent agreement with both theoretical models and values published for a well-separated part of the $_3R$ branch.

Introduction

Despite generally unfavorable excitation conditions, surface-wave overtones, whose eigenfunctions sample the Earth deeper than fundamental modes of similar periods, have long been recognized as powerful tools for the investigation of crust and upper mantle structure. Kovach and Anderson [1964] used them in the 5-50 s range to study the low-velocity zone; later Cara [1973, 1978] and Cara et al. [1980] used mostly $_1R$ and $_2R$, in the 30-70 s range to investigate lateral heterogeneity, Nolet [1975] identified up to $_6R$ in the 20-50 s range, and Okal and Jo [1983] regionalized the dispersion of prominent $_1R$ phases in the 70-150 s range.

In the 80-150 s range, $_3R$, $_4R$, $_5R$, characterized by a very flat surface motion ellipse, have substantial excitability from sources in the 30-450 km depth range. As a result, the radial (away from the source) components of long-period seismograms from major earthquakes exhibit prominently the successive passages of the corresponding Rayleigh waves (see Figure 1). Unfortunately, group velocity curves for the 3 branches feature considerable overlap around 5.8-6.6 km/s, with the details strongly model-dependent (Figure 2). As a result, it is not possible to identify the individual wavetrain $_pR_q$ of the p-th branch in the q-th passage of the overtone wave; indeed Jobert et al. [1977] identified the phase globally as "X_q". More recently, Roult and Romanowicz [1984] used time variable filtering to identify normal modes from fundamentals to 3rd overtones.

The purpose of this paper is to report on methods of extraction of individual overtone branches, and to present preliminary results of an ongoing investigation of their dispersion.

Traveling Wave Approach

Since group velocities overlap at similar frequencies, the classic method of time variable filtering [Landisman et al., 1969] fails if several branches are significantly excited. In this respect, Roult and Romanowicz were successful in extracting the $_3R$ branch only up to $T=114$ s, as expected from Figure 2. We refer the reader to the spatial filtering method of Cara [1973; 1978]: using a number of stations aligned with an epicenter, it maps the energy contained in the U-C plane, and thus efficiently separates branches. For higher overtones at very large wavelengths, it becomes difficult to identify suitable alignments of stations. In order to use the high-quality data from the sparser GDSN Network, we relax the constraint of alignment with the epicenter, and further correct for the azimuthal effect of the focal geometry of the initial phase. Specifically, at each angular frequency ω, the phase φ_j of the Fourier component $a_j(\omega)$ of horizontal ground motion at station j is given by:

$$\varphi_j(\omega) = \varphi_0(\omega) - \pi/4 + (q - 1)\pi/2 - \frac{\omega X_j}{C(\omega)} + \psi_j(\omega), \quad (1)$$

where φ_0 is due to the source time function, $C(\omega)$ is the phase velocity of the mode over the total path X_j traveled to the station (including ellipticity corrections); q is the order of the passage, and $\psi_j(\omega)$ is the phase of the excitation term $[s_R K_0 - l^2 p_R K_2 - i\, l q_R K_1]$ and the notation is that of Kanamori and Cipar [1974]. ψ_j can be computed at all azimuths for a given focal mechanism; its dependence on station azimuth and focal depth is much stronger than on the small variations in C which we are solving for. We used Earth model 1066A to compute excitations and produce synthetic seismograms in order to test the power of the method in separating modes.

We use GDSN data for 4 large earthquakes (1: Colombia, 1979; 2: Santa Cruz, 1980; 3: Banda Sea, 1982; and 4: New Ireland, 1983), with depths between 28 and 450 km, and moments ranging from 2 to 20 times 10^{27} dyn-cm. Figure 3 is an example of U-C diagrams of Santa Cruz records at 99.1 s, stacked for the 4th passage of, respectively, the 3rd and 4th overtones. This particular period was selected to match the mode $_3S_{49}$. It demonstrates the efficient separation of the two branches, and confirms the overlap in group velocity. Tables A1-A3[2] summarize our results for $_3R_4$, $_4R_4$, $_5R_4$.

The formal estimation of the uncertainties accompanying measurements obtained from the two-station method, or by stacking, has traditionally been difficult [Cara, 1978; Mitchell and Yu, 1980]. The error bars listed in Tables A1-A3 represent standard deviations of velocities measured for different events; we use ± 0.02 km/s when only one measurement is available. Within these error bands, there is

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[2] Tables A1-A3 are available with entire article on microfiche. Order from American Geophyscial Union, 2000 Florida Ave., N.W., Washington, D.C. 20009. Document L85-001; $2.50. Payment must accompany order.

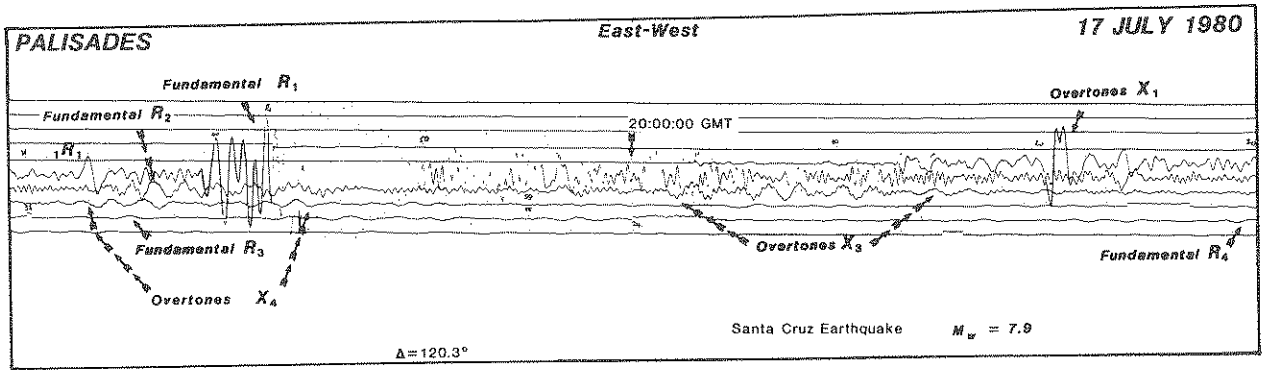

Fig. 1. East-West (away from the source) long period seismogram of Event 2 at Palisades, N.Y., showing the first 4 passages of the fundamental Rayleigh wave R, and of the overtone wavetrain X. The second passage X_2 is lost in the signal of the fundamental R_1, which goes off-scale. Each trace is 1 hr long.

a generally excellent agreement between our measured values and theoretical ones. It starts to deteriorate at longer periods, and for the branch ${}_5R$, which generally contributes less energy to the X phase at these periods.

Normal Mode Approach

An equivalent approach to the retrieval of dispersion, is to extract individual normal modes from Fourier spectra of long sections of seismograms, using stacking to separate branches [Mendiguren, 1973; Gilbert and Dziewonski, 1975]. To compensate for the relatively low number of stations (about 10; Mendiguren used 82), we modify this technique by (a) eliminating most of the fundamental mode signal by winnowing the time series and keeping only sections with group arrival velocities between 5.6 and 6.9 km/s for each subsequent passage; and (b) using a non-linear N-th root stacking technique, designed to emphasize sign (or phase) over amplitude when correlating and stacking elements of a series out of ambiant noise. Specifically, when targetting a particular mode ${}_pS_l$ ($p = 3$, 4, or 5), we compute its excitation

$$E_j = [K_0 s_R \frac{dP_l^0}{d\theta} - K_1 q_R \frac{dP_l^1}{d\theta} + K_2 p_R \frac{dP_l^2}{d\theta}] \qquad (2)$$

at each station j [Kanamori and Cipar, 1974], and form the scalar projector $E_j{}^* a_j(\omega)$, in the vicinity of $\omega = {}_p\omega_l$. If we had full station coverage, this operator would project the spectra a_j on ${}_pS_l$, and eliminate all spurious modes. We enhance the signal-extracting power of the projector by replacing the simple sum by:

$$\left| \left[\sum_{j=1}^{K} \left| E_j a_j(\omega) \right|^{1/N} \exp(i\varphi_j) \right]^N \right| \qquad (3)$$

where K is the total number of stations, and φ_j the phase of the complex number $E_j a_j$ (no sum). This method has proven particularly efficient in extracting small signals from time series of array-type data [Kanasewich et al., 1973]. We use $K = 8$ stations, and $N = 6$ to 10.

After verifying the power of the method using synthetics, we applied it to 20-hr long records of the Santa Cruz event, and extracted 36 modes. Sample spectra are given in Figure 4, and a list of periods in Table A4. The precision on the periods is limited by our frequency resolution, $\delta f = 0.014$ mHz, or ±0.015 km/s in phase velocity. Within this range, our mode picks are in agreement with the phase velocity measurements obtained in the previous section.

Discussion and Conclusion

We have obtained a set of dispersion data for the mantle overtones ${}_3R$, ${}_4R$, ${}_5R$ in the period range 85-140 s. For the surface wave study, the mean deviation from theoretical

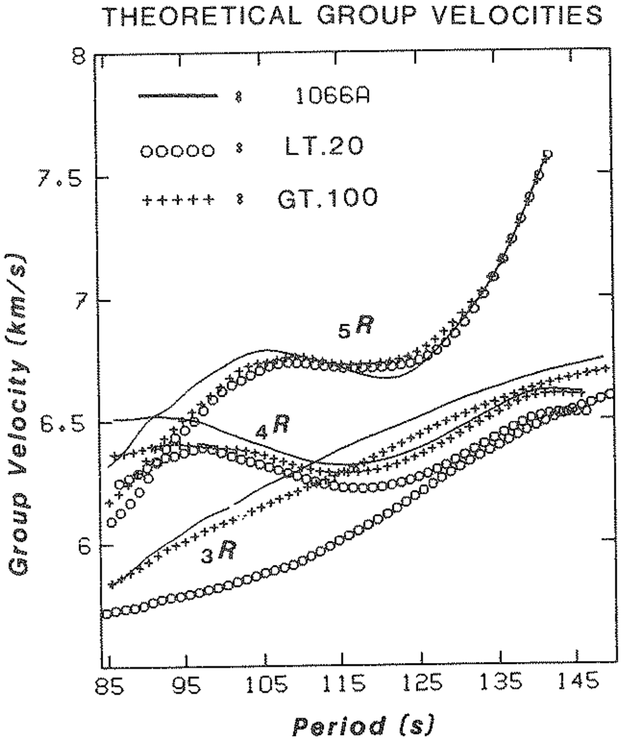

Fig. 2. Theoretical group velocity dispersion curves of the branches ${}_3R$, ${}_4R$, and ${}_5R$, computed for global model 1066A, and two oceanic models (young—LT.20 and old—GT.100; [Mitchell and Yu, 1980]). The details of the inverse dispersion are strongly model dependent, but in all cases, overlap occurs between ${}_4R$ and ${}_5R$ in the 90-100 s range, and between ${}_4R$ and ${}_3R$ in the 105-150 s range.

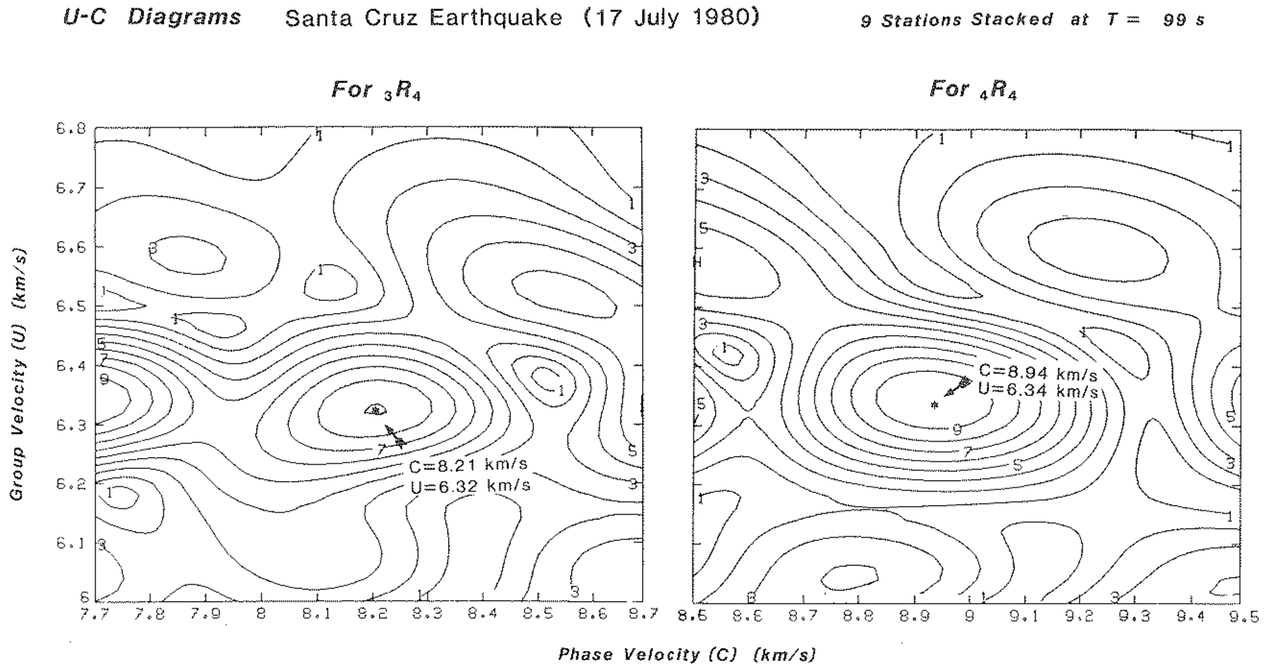

Fig. 3. *U-C* diagrams obtained at a period of 99.1 s, from a stack of 9 GDSN stations for the wavetrain X_4 following Event 2. The left diagram is obtained by stacking for the 3rd overtone $_3R$, and the right one for $_4R$. Numbers on contours are decimal fractions of the maximum amplitude resulting from combining each stack with its Hilbert transform.

results is −0.17% ± 0.33%, −0.49% ± 0.63%, 0.82% ± 0.64%, respectively for the 3rd, 4th and 5th branches. For the mode study, the equivalent numbers are 0.00% ± 0.30%, −0.27% ± 0.21%, and −0.26% ± 0.68%. Given the precision of our measurements (at best 0.15% for each study), these deviations are not significant.

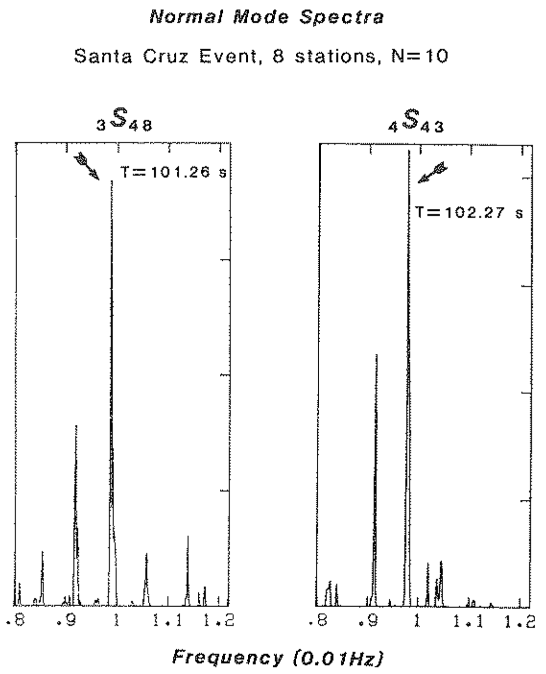

Fig. 4. Example of extraction of normal modes from *N*-th root stacked spectra. The spectrum at left has been stacked for $_3S_{48}$; the one at right for $_4S_{43}$. Note that despite very close frequencies, each mode is absent from the other's stacked spectrum. Vertical scales arbitrary.

Between 114 s and 97 s, our measured periods are an average of 0.23% ± 0.26% shorter than reported by Romanowicz and Roult [1984]. This is less than these authors' frequency resolution; thus the difference is not significant. Below 97 s, the agreement starts to deteriorate due to the generally lesser excitation of the branch.

The agreement obtained with model 1066A is not surprising, since 1066A was obtained from a dataset including observations of overtones. We have embarked in a systematic compilation of their observed periods and attenuations in a variety of geometries, which will improve considerably our resolving power, both for global and regional Earth structure.

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