

Chapter 3

THE ESHELBY-SCHOECK VISCOUS DISLOCATION DAMPING MECHANISM APPLIED TO THE STEADY-STATE CREEP OF ICE

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Introduction

Schoeck (1) has pointed out that in suitable crystals the stress field about a dislocation can produce a region of stress-induced order. Because of the resultant lowering of energy this ordered region is bound to the dislocation, and the dislocation can move only if diffusion processes permit the ordered region to drift. Eshelby (2) greatly refined Schoeck's analysis and generalized it to include the motion of any dislocation moving through a visco-elastic material that can be characterized by the anelastic equations of Zener's standard linear solid (3).

Internal friction experiments have shown that ice is a standard linear solid with a simple damping peak. The origin of the anelasticity in ice is a stress-induced reorientation of the hydrogen atoms about the oxygen atoms. It is natural therefore to see how well the Eshelby-Schoeck dislocation damping mechanism can account for the steady-state creep of ice. In the next section we carry out some calculations to this end.

Eshelby-Schoeck Mechanisms

Eshelby (2) has shown that at moderate stresses a dislocation acted upon by a force σb per unit length (where σ is the shear stress on the slip plane and b is the length of the Burgers vector) moves at a velocity v given by

$$\sigma b \approx Av \tag{1}$$

where

$$A = \frac{\delta\mu}{4\pi^2} \tau v$$

Here μ is an average shear modulus, τ is the average relaxation time of the anelastic material, and δ is the maximum log decrement of the internal friction peak. Let us assume that in ice the Eshelby-Schoeck mechanism is the rate-controlling process for dislocation motion. We have developed (4) a steady-state creep equation that is applicable if the motion of an individual dislocation is controlled by an equation of the form of Eq. 1. If Eq. 1 is substituted into Eq. 6 of Weertman (4), the following expression is found for the rate K of creep in shear when a shear stress σ is applied on the slip plane:

$$K \approx \frac{4\sigma^3}{\mu^2 A} = \frac{16 \pi^2 \sigma^3}{\delta \tau \mu^3} \quad (2)$$

Table 1 shows values of the creep rate calculated from Eq. 1 with the temperature set at -2°C and the shear stress at 1 bar (10^6 dynes/cm²). The experimental shear modulus of 3×10^{10} dynes/cm² and Schiller's determination (5) that $\delta = 0.03$ were used. At -2°C Schiller determined a relaxation time of 1.5×10^{-5} second, a value which is close to that determined from dielectric measurements (6).

Table 1. Creep Rate at 1 bar and Temperatures near -2°C

K_{theory}	$K_{\text{easy glide}}$
$1.3 \times 10^{-5}/\text{sec}$	$1.5 \times 10^{-6}/\text{sec}$ Butkovich and Landauer (7)
	10^{-6} to 10^{-5} Steinemann (8)

Table 1 also lists experimentally determined creep rates of single crystals oriented for easy glide. The data of Steinemann listed in Table 1 were taken from the initial, "well-behaved" part of his creep curves.* Both Steinemann and Butkovich and Landauer observed a power law stress dependence for the creep rate, the power approximating 3. It can be seen that theory and experiment agree within an order of magnitude, which is as well as one can expect in any dislocation theory of plastic deformation.

* The accelerated part of his creep curves can be explained if most of the glide becomes concentrated in a few slip bands. His photograph seems to show that such a concentration does occur.

Polycrystalline Ice

Butkovich and Landauer (7) found that the creep rate ($\sim 10^{-8}$ /sec) of polycrystalline ice tested in shear at a stress of 1 bar is a factor of 100 lower than the value listed in Table 1. Both Steine-
mann (9) and Glen (10), using a uniaxial compressive stress of 1 bar, found the creep rate of polycrystalline ice to be about 1,000 times lower ($\sim 10^{-9}$ /sec). The factor of 10 difference between the shear and compressive tests on polycrystalline ice can, of course, be explained by Nye's analysis of creep under complex stress (11). We believe that the difference in the creep rates of polycrystalline ice and of single-crystal ice oriented for easy glide can be accounted for by the fact that the resolved shear stress on the basal plane of any grain in a polycrystalline mass is less than the applied stress. For example, in the case of a face-centered-cubic metal that is made to creep under uniaxial compression, so many glide systems occur that such a grain has an active glide plane whose resolved shear stress is close to the maximum possible value $\frac{1}{2}P$, where P is the compressive stress. The strain produced across the slip plane also must be resolved into a compressive strain. Hence the creep rate for uniaxial compression or tension will be given by Eq. 1 if P is substituted for σ and the right-hand side is divided by 16.

Ice has only one set of slip planes, the basal planes. Hence in a great many grains of a polycrystalline mass subject to compression, the resolved shear stress on the slip plane is very much smaller than the maximum value of $\frac{1}{2}P$. If θ is the angle that the normal to the slip plane makes with the direction of compression, the resolved shear stress will be* $P \cos \theta \sin \theta$. The resolved creep rate also is multiplied by a factor $\cos \theta \sin \theta$. If we assume that each grain supports an equal share of the total load and that each grain creeps essentially independently of all others, the average compressive creep rate ϵ is given by the following equation:

$$\epsilon = \frac{4P^3}{\mu^2 A} \frac{\int_0^{\pi/2} (\cos \theta \sin \theta)^4 \sin \theta \, d\theta}{\int_0^{\pi/2} d\theta} = \frac{4P^3}{62\mu^2 A} \quad (3)$$

For $P = \sigma$, this equation gives a creep rate 62 times smaller than the creep rate of Eq. 2. We need, however, a factor

* The slip direction on the slip plane is in the direction of the maximum shear stress. See Kamb's discussion on slip direction (12).

closer to 1,000 to explain the difference between creep in easy glide and uniaxial compression creep of polycrystalline ice. It was assumed in Eq. 3 that each grain contributes an equal share to the supporting of the total load across a cross section. It is to be expected, however, that the more easily deformed grains will support less than their share of the total force and thus throw the burden of the load onto the less favorably oriented grains. The creep rate of the more easily deformed grains will be decreased, whereas the creep rate of the less favorably oriented grains will be higher. Since the major portion of the total creep in Eq. 3 comes from the favorably oriented grains, it is to be expected that the over-all creep rate will be reduced. A reduction by a factor of 2 to 3 in the stress acting on the favorably oriented grains would reduce the creep rate of Eq. 3 by a factor of the order of 10. This factor combined with the factor 62 appearing in Eq. 3 would give a total reduction compared to Eq. 2 of the order of 600. We feel it is thus possible to account for the great difference in the creep rate of single-crystal and polycrystalline ice.

Dislocation Climb

Another process that might control the creep of ice is the dislocation climb mechanism. In this circumstance the creep rate under a shear stress σ of a crystal oriented for easy glide is given by Weertman (13):

$$K = \frac{3\sqrt{6} \pi^2 \sigma^{4.5} D}{\mu^3 k T \sqrt{bM\mu}} \quad (4)$$

where k is the Boltzmann constant, T is temperature, M is the density of Frank-Read dislocation sources ($M^{1/2} = 0.5N^{3/4}$, where N is the dislocation density of a three-dimensional dislocation network in a well-annealed crystal), and D is the coefficient of self-diffusion. The self-diffusion of both H and O in ice (actually of O^{18} and deuterium) has been measured by Thürkauf (14) at temperatures around -1.5 to -2°C . The diffusion coefficients for both these elements are identical and equal to 10^{-10} cm²/sec. If this value of D is used in Eq. 4, and if it is assumed that for a well-annealed ice crystal $N = 10^6$ /cm², it is found that the creep rate under a 1-bar shear stress is approximately 1.7×10^{-5} /sec. This rate is about the same as that calculated from the Eshelby-Schoeck mechanism. Thus it is possible that the climb mechanism can account for the observed creep rate.* However, the

* In the discussion at the Chamonix meeting we had erroneously stated that it could not.

experimental data seem to show that a third-power stress dependence usually occurs, an observation that would disagree with the 4.5 power of Eq. 4. (However, Glen (10) found that if he subtracted a transient portion of his creep curves from the total creep curve, the steady-state creep has a 4.2 rather than a 3.2 power dependence.) Unfortunately, the self-diffusion has not been measured over a range of temperature, and the activation energy of diffusion is not known. If we assume that the value of the constant D_0 in the equation $D = D_0 \exp(-Q/kT)$ is about $0.1 \text{ cm}^2/\text{sec}$, we find that the activation energy Q is 11,000 cal/mole, which is close to values reported for the creep of ice. It may not be possible, therefore, to use the self-diffusion activation energy to rule out a climb mechanism.

Discussion and Conclusion

It appears that the Eshelby-Schoeck mechanism can account very well for the creep of ice. However, it is not possible at present to rule out dislocation climb as a rate-controlling mechanism. It should be emphasized that Eshelby's analysis enables one to obtain an expression for the dislocation velocity which contains only experimentally measurable terms. Likewise, the creep equation in which this velocity is used also contains only experimentally measurable terms. There is no need to guess such things as the value of dislocation density. The theoretical values should be good to an order of magnitude.

It appears probable that the wide difference between the creep rates of single crystals and of polycrystalline ice can be explained by geometry effect. The activation energy of creep should be the same as that associated with the internal friction peak if the Eshelby-Schoeck mechanism is rate controlling. Most of the creep evidence today seems to favor a value near 14,000 cal/mole, a value which was first suggested by Landauer. Internal friction experiments as well as dielectric experiments also give activation energies in this neighborhood. It is known from experiments with D_2O ice (6) that the relaxation time of this ice is greater by a factor near $\sqrt{2}$. Hence D_2O ice should creep at a rate slower by this factor than that of ordinary ice. Tests carried out to check this prediction (15) show that D_2O ice does creep more slowly than ordinary ice by amounts of the order ranging from $\sqrt{2}$ to 2. Movement measurements of low-angle boundaries in ice by Higashi and Sakai (16) have shown that dislocations do move*

* Higashi and Sakai mentioned that they will publish a dislocation mechanism which accounts for their results. It will be interesting to see, when more details of this work is available, if Eshelby's equation predicts their dislocation velocities.

by an activated process with an energy in the region of 14,000 cal/mole and that the velocity is proportional to the stress.

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