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CREEP DEFORMATION OF ICE

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INTRODUCTION

Knowledge of the creep properties of ice often is needed in field and theoretical studies of the flow and deformation of glaciers, ice shelves, and ice sheets. Recently many of the icy moons in the solar system have become much better known as a result of the successful deep-space probe missions. An understanding of the physical processes that take place within them requires information about the plastic deformation behavior of ice. Unlike ice within glaciers and ice sheets, the crystal structure of ice deep within an icy satellite may be one of the high-pressure polymorphs rather than that of the more familiar hexagonal ice Ih. At the present time, there exists a great deal of creep data on ordinary ice Ih but very little on other forms of ice.

The aim of this article is to present our basic knowledge on the creep properties of ice. This includes information derived from laboratory experiments as well as gleaned from field measurements made on glaciers, ice sheets, and ice shelves. No attempt is made here to review in a complete manner all the literature on this subject. However, new results from recent papers on ice creep that are not covered in earlier and more complete reviews are included. The reader is referred to the reviews of Hooke (1981; called review H hereafter), Paterson (1977; called review P), Glen (1974, 1975; called review G), and Weertman (1973; called review W) for a more complete coverage of the earlier literature. Review H compares experimental results on ice creep with field measurements on glaciers and other large bodies of ice. Review P summarizes closure data on boreholes in the Antarctic and Greenland ice sheets and Canadian ice caps. Review G covers most aspects of the mechanics and physics of ice. A tutorial-type review on polycrystalline ice has been given by Mellor (1980; called review

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M). Hooke et al (1980) have reviewed the research that still needs to be carried out on the mechanical properties of ice, rather than what has been accomplished up to now. The book by Hobbs (1974) on the physics of ice also covers ice creep. Ice flow law determinations made from measurements of the increase with time of the tilt of vertical boreholes in glaciers are summarized by Raymond (1980) and by Paterson (1981). Goodman et al (1981) have reviewed creep data and creep mechanisms of ice for the purpose of making creep diagrams (deformation maps).

Theoretical explanations of the creep process are given in this article, but in only limited detail. Obviously, the degree of risk involved in extrapolating creep results outside the stress-strain-temperature-time region in which measurements actually were made is a function of how well the creep processes are understood.

Unless otherwise stated, the word "ice" in this review means ordinary ice Ih that exists under ambient pressures and whose hexagonal crystalline structure gives rise to the classic form of snowflakes. The hexagonal crystal form is also the origin of the large anisotropy of the creep properties of ice. Slip occurs readily across the basal planes (normal to the crystal c-axis) of an ice crystal. If stress of magnitude sufficient to produce plastic deformation is applied to an ice single crystal, the ice crystal is "softer" if the applied stress has a shear component resolved across the basal planes that is relatively large. If the resolved basal shear stress is relatively small, the ice crystal is "harder" and the amount of plastic deformation in a given time period is smaller. In the former situation the ice crystal is in an "easy" glide orientation in which lattice dislocations are moved easily across the basal slip planes to produce slip. In the latter situation the ice crystal is in a "hard" glide orientation because deformation requires slip on nonbasal planes. The deformation of the grains of polycrystalline ice generally requires a hard-glide component and thus polycrystalline ice usually is "hard" too.

Our review is restricted to the creep properties of ice of grain size sufficiently large (>0.5 mm) that deformation occurs primarily by dislocation motion. Only the creep observed at moderate to large strains (greater than, say, 0.1%) is considered. Excluded are small-strain anelastic deformation and creep produced by the mechanisms of the creep of very fine grain material.

TIME DEPENDENCE OF CREEP STRAIN FOR CONSTANT STRESS TESTS

Two types of curves of creep strain ε versus time t are found for ice specimens made to creep by application of a constant stress. These are shown in Figure 1. Both polycrystalline ice and ice single crystals in a hard

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orientation usually exhibit a creep rate $\dot{\varepsilon} \equiv d\varepsilon/dt$, which decelerates as the creep strain increases (see reviews G and W). At larger creep strains the creep rate approaches a constant, or quasi-constant, steady-state value. The creep rate of ice single crystals that are in an easy-glide orientation usually accelerates toward the steady-state creep rate that exists at the larger plastic strains (see reviews G and W).

The two types of creep curves shown in Figure 1 also describe the creep behavior of other crystalline material. Germanium and silicon (Alexander & Haasen 1968), as well as some alloys that are called class I alloys by Sherby & Burke (1967) and class A (Alloy type) by Yavari et al (1981), have creep curves that accelerate into the steady-state region. (Class I/A alloys have a power-law creep exponent n with a value $n \simeq 3$.) The creep curves of pure metals and alloys called class II by Sherby & Burke [pure metal and class II alloys are called class M (Metal type) by Yavari et al] always follow a decelerating approach of the creep rate into steady state. But some class I/A alloys also show this latter behavior. (Class II/M metals and alloys have a power-law exponent of $n \simeq 4$ to 5.) In the case of rocks and rock minerals, our knowledge of the behavior in the transient-creep region is somewhat uncertain because of the experimental difficulties in applying pure hydrostatic pressure in a test run to prevent microcracking of a specimen. Data reviewed by Carter & Kirby (1978) showed decelerating transient creep.

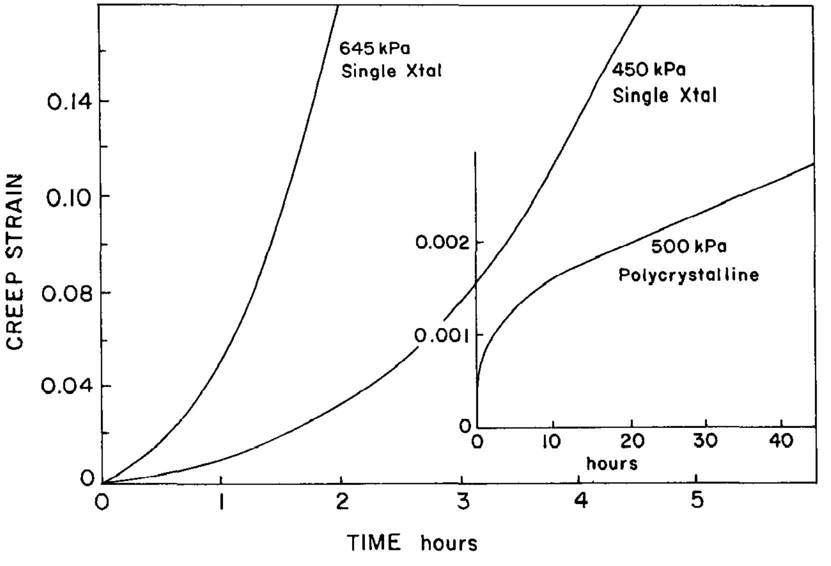

Figure 1 Creep strain versus time under constant stress for polycrystalline ice at −7°C (after Duval & Le Gac 1980) and easy-glide single crystals at $\simeq -10.5$°C (after Ramseier 1972).

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However, studies on olivine (Durham et al 1977, 1979), which has $n \simeq 3$, indicate that the transient creep could be of the accelerating type.

Ice is somewhat unusual in that both accelerating and decelerating transient creep are found for it. But metal alloys that are on a borderline between class I/A and class II/M behavior also exhibit both types of transient-creep behavior (Oikawa et al 1977). [In the alloys examined by Oikawa et al, as well as those looked at by Yavari et al (1981), a change from $n \simeq 3$ to 5 could be accomplished by altering the magnitude of the applied stress. Vagarali & Langdon (1982 and private communication) have shown even more clearly for a hcp Mg-Al alloy the switch from accelerating to decelerating transient creep, the increase of n from 3 to 4, and the existence (or nonexistence) of the upper yield point with increase of stress.]

There is a simple qualitative explanation for the creep curves of Figure 1. Material in which creep starts out with an accelerating rate presumably are those in which the initial dislocation density is small and in which dislocation glide motion is hindered by any number of drag mechanisms. Dislocation multiplication, which requires dislocation motion, is also hindered. The creep rate is given by the equation

$$\dot{\varepsilon} = \alpha \rho b v, \tag{1}$$

where v is the average dislocation velocity, b is the length of the burgers vector, ρ is the dislocation density, and α is a dimensionless geometric factor whose exact value, usually of order of magnitude one, depends upon the orientation of the slip planes. For drag mechanisms the dislocation velocity, until some critical stress level is exceeded, is given by

$$v = v_0(\sigma b^3/kT) \exp(-Q/kT), \tag{2}$$

where v_0 is a constant of dimension of velocity, σ is the stress, k is Boltzmann's constant, T is the temperature, and Q is the activation energy of the drag mechanism. As the dislocations move, more and more dislocations are created. The dislocation density increases up to the steady-state density given by

$$\rho = \beta \sigma^2/b^2\mu^2, \tag{3}$$

where β is a dimensionless constant whose experimentally determined value (Weertman 1975) is of order 1, and μ is the shear modulus. [At the density given by (3), the internal stress produced by the dislocations is of the same magnitude as the applied stress.] Combining (1), (2), and (3) gives for the steady-state power-law creep rate

$$\dot{\varepsilon} = \dot{\varepsilon}_0(\sigma/\mu)^n(\mu b^3/kT) \exp(-Q/kT), \tag{4}$$

where $n = 3$ and $\dot{\varepsilon}_0 = \alpha\beta v_0/b$. This equation is that of a class I/A alloy.

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An explanation of the decelerating creep rate seen in class II/M material is that the dislocations experience a smaller resistive drag to their glide motion compared with that of their climb motion. Thus when a stress is applied, dislocations multiply almost instantaneously to give a density of the order of (3). A large amount of glide motion occurs and produces an instantaneous plastic strain. Because the dislocation morphology that exists after the "instantaneous" plastic deformation presumably is not the most stable dislocation arrangement possible, the dislocations start to rearrange themselves into a more stable pattern. The creep rate slows down until steady-state creep is attained. The creep rate presumably is controlled by dislocation climb, which in turn is controlled by self-diffusion. At high stresses and/or lower temperatures and moderate stresses, it is likely to be controlled by dislocation pipe self-diffusion.

One difficulty remains to be resolved for this simple picture of climb-controlled creep for class II/M material: If no ad hoc assumptions are made in the dislocation models used to calculate the creep rate, the power-law exponent is equal to $n = 3$ (Weertman 1975) when climb is controlled by bulk diffusion, rather than the experimentally observed $n = 4$ to 5. [When pipe diffusion, i.e. diffusion down dislocation cores, controls the climb, the value of n is increased by 2 to $n = 5$ (Robinson & Sherby 1969, Evans & Knowles 1977, Langdon 1978, Langdon & Mohamed 1978, Sherby & Weertman 1979, Spingarn et al 1979). However, experimentally $n \simeq 6$ to 7 in this situation.] An experimental determination that $n = 3$ is, therefore, no guarantee that a dislocation drag mechanism controls the creep rate. But a value of $n = 3$, coupled with the observation that the creep rate accelerates during the transient period, should give strong support that such a mechanism does control the creep rate. Because the steady-state creep of ice single crystals in an easy-glide orientation has an exponent $\simeq 2$–3.9 (see Table 1), and because of results found on the motion of individual dislocations that are considered later, it appears likely that a dislocation drag process does control the creep of these crystals.

The implication of this explanation of accelerating creep is that if the strength of the drag process could be reduced sufficiently the transient behavior of easy-glide crystals should change from accelerating to decelerating behavior. Doping ice crystals does increase the dislocation velocity and decrease the drag force (Mai et al 1978). Riley et al (1978) have made the interesting observation that NaCl-doped easy-glide single crystals have a decelerating transient creep. (They did not determine the value of n because the steady-state creep region was not reached in their experiments.)

The value of n for polycrystalline ice and ice single crystals of hard orientation also is approximately equal to three (see Table 1). The rate-

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Table 1 Power law exponent n and creep activation energy Q

n	Q (kJ mole^{-1})	Comment	Reference
Polycrystalline Ice below −10°C			
3.1	59.9	—	Ramseier (1972)
3	59.9	Frazil ice	Ramseier (1972)
2.5	59.9	Columnar ice	Ramseier (1972)
3.1	77.9	—	Barnes et al (1971)
3.1–3.6	83.8	—	Steinemann (1958) with reanalysis by Barnes et al (1971)
—	50.3	Friction exp.	Bowden & Tabor (1964)
3.5	41.9	—	Bender et al (1961)
3.2	41.9	D_2O ice	Bender et al (1961)
3.5	44.8	—	Mellor & Smith (1967)
—	68.7	—	Mellor & Testa (1969a)
—	64.9	Columnar ice	Gold (1973)
2.9–3.05	67	Columnar ice	Sinha (1978, 1982)
3	—	Creep rate of floating ice shelves	Thomas (1971)
3	54	Secondary creep rate determined from closure rates in boreholes	Paterson (1977)
3	≃74	Our estimate of Q from their data	Russell-Head & Budd (1979)
—	78	Antarctic ice	Duval & Le Gac (1982)
Polycrystalline Ice above −10°C			
3.2	134	—	Glen (1955)
2.8–3.2	134	—	Steinemann (1958) with reanalysis by Barnes et al (1971)
3.2	122	—	Barnes et al (1971)
—	≃168	—	Mellor & Testa (1969a) with review W analysis
3	—	Glacier tunnel closing	Nye (1953)
3	≃200	Our estimate of Q from their data	Russell-Head & Budd (1979)
3	—	Triaxial stress tests	Duval (1976)
Single crystals in easy glide			
2.5	—	—	Butkovitch & Landauer (1958, 1959)
4	—	—	Glen (1952)
1.5–3.9	—	—	Steinemann (1954)
1.6	66.2	Bend test	Higashi et al (1965)
—	65.4	−50°C to −10°C	Jones & Glen (1968)
—	39.8	−90°C to −50°C	Jones & Glen (1968)
4	—	−50°C	Glen & Jones (1967)

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Table 1 (*continued*)

n	Q (kJ mole^{-1})	Comment	Reference
$\simeq 2$	59.9	—	Ramseier (1972)
—	—	−20°C to −0.2°C No activation energy change, value not stated	Gold (1977)
1.9	78	−30°C to −4°C	Homer & Glen (1978)
2.9	75	Bicrystals −30°C to −4°C	Homer & Glen (1978)
2	70	−20°C to −0.2°C	Jones & Brunet (1978)
1.8–2.6	81	−5°C to −7°C	Nakamura (1978)
Single crystals in hard glide			
3	—	—	Butkovitch & Landauer (1958, 1959)
—	57.0	—	Ramseier (1967a,b)
—	69.1	—	Mellor & Testa (1969a)

controlling process for this material is not certain. The fact that $n \simeq 3$ lends support to a drag mechanism but, as already mentioned, this value does not necessarily rule out a dislocation climb control mechanism.

CONSTANT STRAIN RATE TESTS

If ice is deformed at a constant strain rate (rather than under a constant stress) a curve of stress versus strain (rather than strain versus time) is obtained. Figure 2 shows the type of stress-strain curve that is found for single crystals of ice in an easy-glide orientation and in some tests on polycrystalline ice (reviews G and W). As the strain is increased, the stress reaches a maximum, "upper yield point" value, falls off, and finally reaches a steady-state value. In the case of ice crystals of hard orientation and in other polycrystalline ice tests, the stress-strain curve obtained in a constant strain rate test is different (reviews G and M, Higashi 1967, Hawkes & Mellor 1972, Shoji 1978). The curve is essentially the same as a metal tested at constant strain rate at a high temperature. The stress increases with strain and reaches a quasi-saturation, quasi-steady-state level. (A slight decrease in stress in high strain rate tests, which seems to be produced by cracking, may occur at large strains.) It should be noted that once the stress reaches a steady-state value the constant strain rate test is the equivalent of a constant stress test when the creep rate has attained a steady-state value (Mellor & Cole 1982).

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A stress-strain curve of the type shown in Figure 2 is readily explained (see reviews G and W and the literature quoted there) if dislocation motion, and thus creep, is controlled by a drag mechanism. In the initial stage of a test there are only a few dislocations present. The strain is almost all elastic. The stress must increase as the elastic strain is increased when the total strain rate is held constant. As the stress increases, the dislocations that are present move faster, multiply more rapidly, and add a greater plastic component to the total strain. Eventually enough dislocations exist that all of the strain rate can be produced by their movement at a smaller average velocity and at a lower stress. Hence the stress falls off to a much lower level.

The shapes of the curves of easy glide for the ice single crystals of Figures 1 and 2 thus can both be explained qualitatively if drag controls the glide motion of dislocations and the creep rate. Similarly, for the case of hard-glide crystals and some polycrystalline ice the fact that the creep curves show deceleration and the stress-strain curve shows no marked upper yield point might imply that dislocation climb, rather than drag, controls the creep rate. If so, it is puzzling why it is so much more difficult to make ice slip on nonbasal planes. We leave this as an exercise for the reader to solve; a second part of the exercise is given when borehole closure results are considered. A partial answer is given later.

Stress-strain curves have also been obtained on columnar ice under application of a stress that increases at a constant stress rate (Sinha 1982). No upper yield point is observed. The stress increases monotonically with

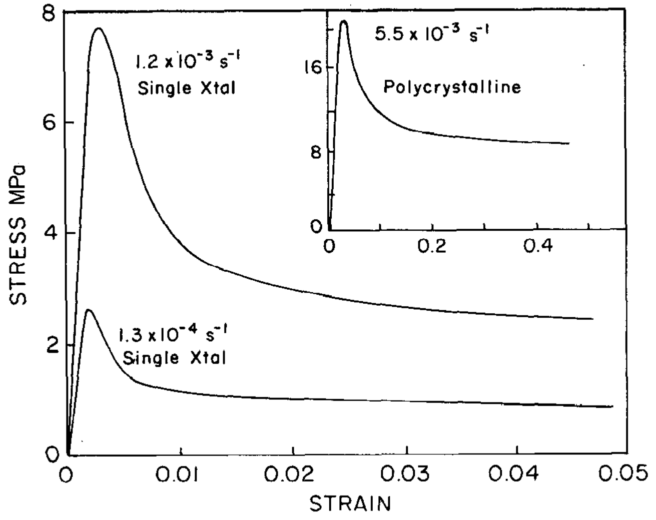

Figure 2 Stress versus strain at constant strain rate for polycrystalline ice at −11°C (after Jones 1982) and easy-glide ice single crystals at −10°C (after Ramseier 1972).

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increasing strain but with a decreasing work-hardening coefficient (defined to be the slope $d\sigma/d\varepsilon$ of the stress-strain curve). Cracking within the ice makes the results at later stages ambiguous.

STRESS DEPENDENCE OF STEADY-STATE CREEP RATE

At moderate stresses, as the reader must have deduced from the discussion of a previous section, the steady-state creep rate of ice is given by the power-law creep equation

$$\dot{\varepsilon} = B\sigma^n \exp(-Q/kT). \tag{5}$$

Here B is a constant. In the glaciological literature this equation is known as Glen's creep law for ice because he showed first that it describes the creep of ice (Glen 1955).

In Table 1 are listed measured values of the power exponent n and the creep activation energy Q obtained from both laboratory experiments and field measurements. Figure 3 is a log-log plot of stress and creep rate for some of these data.

It should be noted in Table 1 and Figure 3 that the power exponent determined in most tests is approximately equal to 3. Also note that the activation energy for creep of single crystals up to the melting temperature (and above $-50°C$) is approximately 60 kJ mole^{-1}. This value is, within experimental error, the same as that found for the self-diffusion of hydrogen and of oxygen in ice (see reviews G and W). Hydrogen and oxygen diffuse at identical rates in ice. The equality of the creep and diffusion activation energies cannot be used as proof that the rate-controlling process is dislocation climb (which in turn is controlled by self-diffusion) because drag mechanisms can also involve the motion of hydrogen in the lattice.

Polycrystalline ice has the same activation energy as single crystal ice at temperatures below $-10°C$, but it has a value somewhat greater than twice this value at temperatures above $-10°C$. The larger activation energy of polycrystalline ice at the warmer temperatures generally is attributed to the softening that is produced by the significant amounts of recrystallization and grain growth that occur simultaneously with the creep deformation. The experiment of Jones & Brunet (1978) on single crystals was done for the express purpose of ascertaining whether there was a true change in activation energy in the dislocation deformation process. The activation energy was observed not to change. The ratio $\dot{\varepsilon}_{-1°C}/\dot{\varepsilon}_{-10°C}$ for polycrystalline ice is 7.6. For single crystal ice the same ratio is 2.9. Thus although the change in activation energy of polycrystalline ice is quite large,

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the maximum increase in the creep rate it produces is relatively small when it is considered that Q enters the creep rate in an exponential term.

It should also be noted in Figure 3 that the creep rate at a given stress is much larger for easy-glide crystals than it is for hard-glide crystals and for polycrystalline ice. The difference is a factor of about 500.

At large stress levels the data of Barnes et al (1971) show that the power

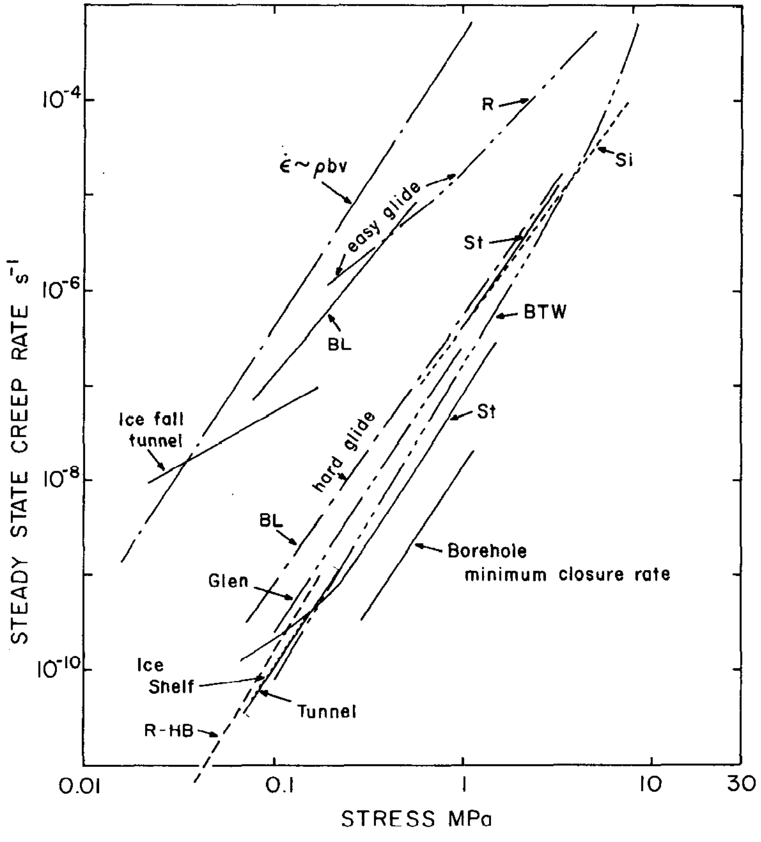

Figure 3 Log-log plot of steady-state or the minimum creep rate versus stress. Creep rate is normalized to $-10°C$, with $Q = 134\ kJ\ mole^{-1}$ for polycrystalline ice above $-10°C$ and $60\ kJ\ mole^{-1}$ below this temperature or the activation energy reported by experimenter. Creep rate and stress are for, or have been changed to, the equivalent of a uniaxial tension or compression test. Data from the following sources. BL: Butkovitch & Landauer (1958, 1959); BTW: Barnes et al (1971); Glen: Glen (1955); St: Steinemann (1954); R: Ramseier (1972); Si: Sinha (1982); R-HB: Russell-Head & Budd (1979); ice shelf: Thomas (1971); glacier tunnel closing (Vesl-skautbre in Norway and Z'Mutt in Switzerland): Nye (1953); ice fall tunnel closing (Austerdalsbre in Norway): Glen (1956); borehole closing (Byrd in Antarctica, Site 2 in Greenland, and Meighen and Devon in Canada): Paterson (1977).

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law breaks down, in agreement with observations on other crystalline material. Over a stress range that includes large stresses, creep data, including those of ice, follow the more general power sinh equation of Garofalo: $\dot{\varepsilon} \simeq [\sinh (\sigma/\sigma^*)]^n$, where σ^* is a constant. The sinh function reduces to its argument when its argument is small, and thus when $\sigma < \sigma^*$ the Garofalo equation reduces to the power-law equation.

Creep data on ice obtained at very low stresses and creep rates, which are not plotted in Figure 3 or tabulated in Table 1, generally indicate that the creep rate is proportional to stress (Mellor & Testa 1969b). The creep rates at a given stress level show scatter between different investigators. Because the creep strains are so small, these data should not be compared with those of Figure 3, which are obtained at much larger strains. In fact, the data of Russell-Head & Budd (1979) shown in Figure 3 obey a third-power relationship, whereas small-strain experiments in the lower part of the same stress region give a power closer to one. The Russell-Head & Budd work is remarkable in that they carried out experimental runs up to two years in time in order to obtain larger strains. [A linear relationship at low stresses and small strain can be accounted for with a grain boundary sliding mechanism, the Nabarro-Herring or Coble mechanism of mass diffusion of point defects between grain boundaries, and a dislocation mechanism using (1) and (2) if there is no significant change in the dislocation density upon and after application of the stress.]

Field Data

In Figure 3 are plotted data obtained from field measurements of the deformation of floating ice shelves, and from the closure of tunnels excavated in glaciers and boreholes drilled in ice sheets and ice caps. An unconfined floating ice shelf deforms in a very simple manner. It thins under its weight in a manner that is exactly equivalent to that of a laboratory sheet specimen that is pulled in two orthogonal directions in biaxial tension. [If the ice shelf is unconstrained in both horizontal directions, the two stresses of the biaxial test are equal to each other. If the ice shelf can spread in only one direction, the two biaxial stresses are not equal to each other. One has a deviatoric value (see a later section) equal to zero.]

The theory for the spreading of ice shelves is particularly simple. Thomas (1971) has used this theory, part of which he has helped develop, to analyze spreading-rate measurements reported from different ice shelves. In Figure 3 is plotted the creep rate stress relationship for ice, which is deduced from his calculations. It agrees reasonably well with the slower laboratory creep data.

Tunnel-closing data have been analyzed by Nye (1953), who developed the theory for tunnel closing. These data also are plotted in Figure 3, and

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agree with Thomas' results and the slower laboratory data. The Nye theory for tunnel closing applies equally well to the closure rate of boreholes drilled into ice. In Figure 3, borehole data from different sources, analyzed by Paterson (1977) using Nye's theory, lie considerably below all the other data. The only difference between the tunnels and the boreholes is that the tunnels were in ice near the melting point and the boreholes were in cold ice. A man or woman could enter the tunnels but a borehole could only accommodate one of their arms or legs. However, the closure curves of the boreholes exhibit a curious behavior. The closure, converted into a true creep rate, generally took place at an increasing rate, although a minimum rate occurred shortly after the start of closure. In other words, the polycrystalline ice around the borehole has a creep curve somewhat like that of Figure 1 for easy-glide ice single crystals. The creep rate plotted in Figure 3 for the borehole data is the minimum creep rate. The creep rate increased by factors of 3 to 10 during the course of closure. Hence the steady-state creep rate for the ice around boreholes should lie much closer to the tunnel-closing data and the ice shelf data. Why this polycrystalline ice from the Arctic and the Antarctic shows this acceleration is the second part of the exercise mentioned earlier.

Field data on the change with time of the vertical tilt of boreholes in glaciers, ice sheets, and ice caps also give information from which the creep properties of ice can be deduced. The creep within a glacier is primarily a shear deformation that increases from the upper surface to the lower surface. Thus a vertical borehole in time is tilted. The amount of tilt increases with depth. Extracting the ice creep properties from these data is made difficult partly because of multi-stress component effects on the creep rate. (These effects are discussed in a later section. The ice fall tunnel line in Figure 3 is an example of such an effect.) The uncertainty in the estimates of the shear-stress magnitude at different depths below the surface is also a problem.

Creep rate data determined from boreholes in glaciers [summarized by Raymond (1980) and by Paterson (1981)] have not been included in Figure 3. However, Table 2 lists values of the stress exponent n and values of the creep rate at 0.1 MPa (creep rate and stress appropriate for uniaxial tension or compression tests) for data reanalyzed by Raymond (1980). The creep rate is normalized again to $-10°C$ with use of the activation energy $Q = 134$ kJ mole^{-1}. The ice of the glaciers is close to the melting point. Also listed in Table 2, for comparison purposes, is the average creep rate at 0.1 MPa of the ice shelf, glacier tunnel closing, Russell-Head & Budd, and Barnes et al data of Figure 3. It can be seen that the exponent value given by the tilt results is approximately three. The absolute values of the creep rates are approximately the same as the smaller creep rate lines of

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Table 2 Creep data for borehole tilt measurements after reanalysis by Raymond (1980)[a]

Glacier	n	$\dot{\varepsilon}$ ($s^{-1} \times 10^{10}$)	Source
Salmon (British Columbia, Canada)	2.8	1.3	Mathews (1959)
Athabasca (Alberta, Canada)	4	0.9	Paterson & Savage (1963)
Athabasca	3.6	0.9	Raymond (1973)
Blue (Washington, USA)	3.3	0.9	Shreve & Sharp (1970)
Figure 3 data (slower creep rate data)	3	1	—

[a] Creep rate normalized to $-10°C$, with $Q = 134$ kJ mole^{-1}. Creep rate is for the equivalent of a longitudinal strain rate under a uniaxial compressive or tensile stress of 0.1 MPa.

Figure 3. These lower creep rates probably are the best ones to use in theoretical calculations of the flow of glaciers and ice sheets. Goodman et al (1981) have used a creep diagram and the Byrd borehole data to successfully account for the horizontal ice movement measured at Byrd station, Antarctica.

Complications

The steady-state creep of ice actually is a bit more complicated and not as neat and tidy as has been indicated so far. Curves of creep rate versus creep strain at constant stress in the so-called steady-state region generally show a minimum in the creep rate, which is followed by somewhat higher strain rates at larger strains (review M, Mellor & Cole 1982). Budd (unpublished data) and Mellor (review M) have pointed out that the minimum creep strain under a fixed stress is reached at longer times, but at the same strain, as the test temperature is lowered. They also point out that some apparent conflicts in reported experimental and field results may simply be a consequence of not taking the position of the minimum creep rate into account. That is, one experiment may have been carried out on one side of the minimum and another on the other side. Steady-state creep rates should be regarded as quasi-steady-state creep rates. Figure 4 illustrates one rather extreme example of a large increase in creep rate of polycrystalline ice that follows the minimum creep rate. This increase is apparently produced by the development of a fabric, with the ice crystals becoming more favorably oriented for creep. The lower curve exhibits irregularities in the strain rate of another sample that is produced by simultaneous recrystallization. The increase in the activation energy of polycrystalline ice, which is produced presumably by recrystallization, has already been mentioned. Detailed

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studies of the development under stress of crystal orientation fabrics have been carried out by Kamb (1972), Budd (1972), and Wilson & Russell-Head (1982).

Because ice crystal orientation fabrics develop within ice sheets at different depths, the creep properties of ice in large ice masses can be a strong function of the creep flow itself. Russell-Head & Budd (1979) have analyzed the shear in creep from borehole tilt measurements in the Law Dome, Antarctica. They find that the shear-strain rates are anomalously large because of fabric development. At intermediate depths, a higher fraction of the ice has its *c*-axis oriented near a vertical direction. Analysis of the ice fabrics of the Camp Century and Dye 3, Greenland, borehole ice (Herron 1982, Herron et al 1982) and the Byrd Station, Antarctica, borehole ice by Gow & Williamson (1976) reveals a strong increase with depth of the fraction of ice with a near-vertical *c*-axis.

Creep experiments carried out by Shoji & Langway (1982) on the strongly oriented polycrystalline ice (Herron et al 1982) obtained from the Dye 3 borehole close to the bottom of the 2000-m core, as well as ice from

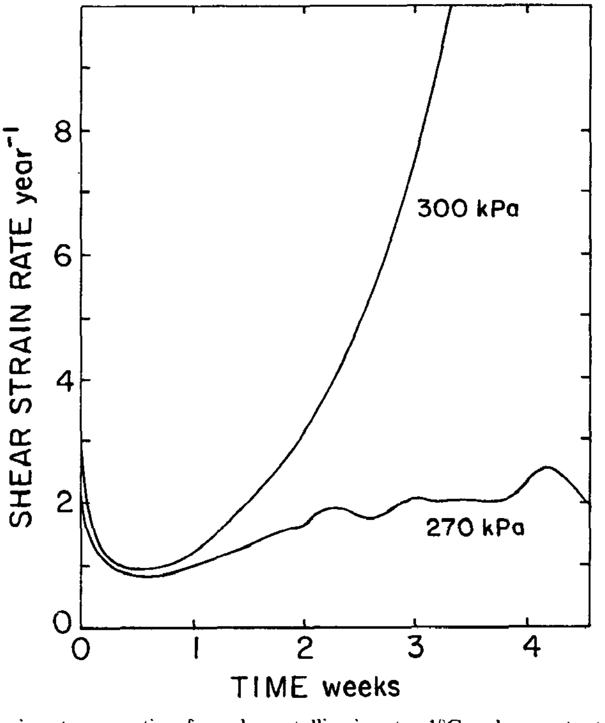

Figure 4 Strain rate versus time for polycrystalline ice at -1°C under constant stress that is undergoing recrystallization and fabric development (after Duval 1981).

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the Barnes Ice Cap margin (Baker 1981), gave creep rates that are a factor of four larger than those of the ice shelf, glacier tunnel, R-HB, and BTW lines in Figure 3 of randomly oriented polycrystalline ice. The experiments were carried out in a field laboratory immediately after the ice was brought up from the borehole. The same factor of four enhancement was found by Russell-Head & Budd (1979) in the creep rate of the strongly oriented ice that occurs at intermediate depths in the Law Dome boreholes. Russell-Head & Budd determined the creep rates from borehole tilt measurements as well as from laboratory experiments. It appears reasonable from these results to use an enhancement factor of four in ice-modeling work wherever strongly oriented ice is expected or is known to occur.

An implicit assumption in the plots of Figure 3 for polycrystalline ice is that there is no significant grain size effect on the creep rate. For large grain material a grain size effect is not expected. However, the experimental

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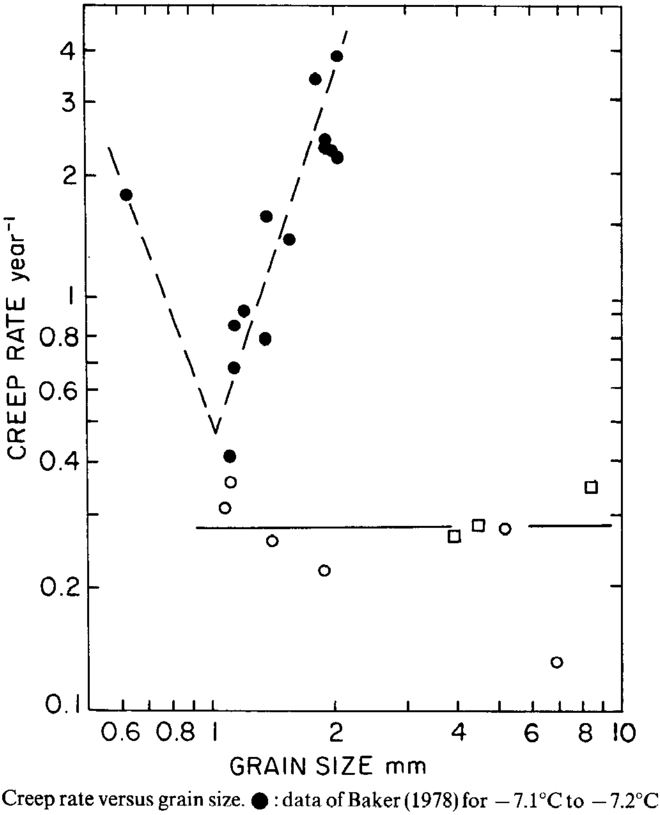

Figure 5 Creep rate versus grain size. ●: data of Baker (1978) for −7.1°C to −7.2°C at stress of 560 kPa; ○, □: data of Duval & Le Gac (1980) for −7.0°C to −7.2°C at stress of about 500 kPa. The open squares are for Antarctic ice.

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evidence on grain size effect is sparse and somewhat in conflict. Baker (1978, 1981) reported that there is a strong grain size dependence for grain sizes between about 0.6 mm to 6 mm. His data, some of which are plotted in Figure 5, show that the creep rate goes through a minimum at a grain size near 1 mm. However, Duval & Le Gac (1980) failed to find a grain size dependence for steady-state creep in their tests on specimens of grain sizes between about 1 mm and 10 mm, although a weak dependence was found for transient creep.

Anomalously large creep rates (up to almost an order of magnitude larger) have been measured in experiments carried out on polycrystalline ice whose temperature is extremely close to the melting temperature (within 0.01°C) (Barnes et al 1971, review H, Duval 1977, Budd, unpublished data). Duval (1977) looked at this problem by measuring the creep rate of ice that is close to its melting temperature as a function of the water content of a sample. He found that the creep rate tripled when the water content increased from 0.01% to 0.8%. The variation of creep rate with water content was approximately linear. Duval attributed his results to the fact that samples with different water contents had different salt contents and different melting temperatures. Impurity-doped ice is known to creep appreciably faster than pure ice (reviews G and W).

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CREEP UNDER MULTI-COMPONENT STRESS

In glaciers, ice sheets, and ice shelves, and around boreholes and tunnels excavated in them, the stress state is almost always a multi-component stress state. Thus in any complete formulation of ice flow in these ice bodies a more general form of the creep equation than the one-component stress equation (4) is needed.

Suppose acting on isotropic, polycrystalline ice there are more than one stress components σ_{ij} that have nonnull values. More than one of the tensor strain rate components $\dot{\varepsilon}_{ij}$ in steady-state creep will have nonnull values too, except for the case of pure hydrostatic pressure. Nye (1953) obtained for an incompressible solid a generalized creep equation for multi-stress by using the analogue of plasticity theory's Lévy-von Mises relations. Since the material is considered to be isotropic, the creep equation can be a function of the three stress invariants. The first stress invariant is the hydrostatic pressure P given by

$$P = -(\sigma_{11}+\sigma_{22}+\sigma_{33})/3 = -\sigma_{ii}/3. \tag{6}$$

The effect of P is taken into account in (5) by rewriting this equation as

$$\dot{\varepsilon} = B\sigma^n \exp(-Q/kT) \exp(-VP/kT), \tag{7}$$

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where V is the activation volume for creep, a quantity that can be determined experimentally.

Since creep does not take place under pure hydrostatic pressure in fully dense material, it is convenient in the above equations to use the deviatoric stress components σ'_{ij} defined by

$$\sigma'_{ij} = \sigma_{ij} + \delta_{ij} P, \tag{8}$$

where $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$. (Note that $\sigma'_{ij} = 0$ for pure hydrostatic pressure.)

The second stress invariant τ can be defined by

$$2\tau^2 = \sigma'_{ij}\sigma'_{ij}, \tag{9}$$

where the right-hand side is summed over all the deviatoric stress components. (When $i \neq j$, σ'_{ij} and σ'_{ji} are each counted.) The analog of the Lévy-von Mises relations requires for any two stress components and the corresponding strain rate components that

$$\sigma'_{ij}/\sigma'_{mn} = \dot{\varepsilon}_{ij}/\dot{\varepsilon}_{mn}, \tag{10}$$

where $\dot{\varepsilon}_{ij}$ are the tensor strain rate components. [The effective strain rate $\dot{\varepsilon}_{\mathrm{eff}} = (\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij}/2)^{1/2}$ is also invariant. For an incompressible solid in plastic deformation, $\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33} = 0$.]

Suppose in a uniaxial tension or compression test the relationship between longitudinal creep rate $\dot{\varepsilon}$ and the tensile or compressive stress is given by a general equation

$$\dot{\varepsilon} = f(\sigma), \tag{11}$$

where $f(\sigma)$ is an arbitrary function. Power-law creep is one special case. [If pressure $P \neq 0$, its effect is taken into account through the expression $\exp(-PV/kT)$ in the function f.] Equations (10) and (11) are satisfied if

$$\dot{\varepsilon}_{ij} = f(\sqrt{3}\tau)(\sqrt{3}/2\tau)\sigma'_{ij}. \tag{12}$$

(Note that $\sigma = \sqrt{3}\tau$ for the uniaxial stress situation. If σ_{11} is the tensile stress σ, then $\sigma'_{11} = 2\sigma_{11}/3$, $\sigma'_{22} = \sigma'_{33} = -\sigma_{11}/3$. Also, $\dot{\varepsilon} = \dot{\varepsilon}_{11} = -2\dot{\varepsilon}_{22} = -2\dot{\varepsilon}_{33}$.) Equation (12) reduces to (11) for uniaxial tensile creep, and the transverse strain rate components have their correct values, too. Thus (12) is a reasonable generalization for creep under multi-component stress for isotropic material, and has been used extensively in analyses of glacier flow problems. For power-law creep the function f is

$$f(\sigma) = B \exp(-Q/kT) \exp(-PV/kT)\sigma^n. \tag{13}$$

The third stress invariant, which is $\sum\sigma'_{ij}\sigma'_{jk}\sigma'_{ki}/3$, has been considered for

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equations of multi-component creep (Glen 1958). It has not been clearly demonstrated that it is necessary to include this invariant in the creep equation for isotropic polycrystalline ice (see review G). Creep experiments of Duval (1976) for polycrystalline ice under triaxial stress conditions strongly support Nye's use of the second stress invariant. However, the third stress invariant appears to be needed to account for the creep properties of anisotropic polycrystalline ice (Budd, private conversation, Lile 1978).

Nye (1953) used the above equations to calculate the expected closure rate s (s = radial wall velocity divided by wall radius) of round tunnels and boreholes in ice bodies. For the case of power-law creep with $n = 3$, Nye's results reduce to

$$s = B \exp(-Q/kT) \exp(-PV/kT)P^3/6, \tag{14}$$

where P is the overburden pressure. The value of τ given by (9) is $\tau = -P/3$. Since $\sigma = \sqrt{3}\tau$ in a uniaxial stress, it is necessary in comparing uniaxial creep date with closure data to set $\sigma = -P/\sqrt{3}$ and $\dot{\varepsilon} = -2s/\sqrt{3}$. This is done in Figure 3.

A very striking example of the effect that one stress component has on the strain rate of a different component is the closure rate, measured by Glen (1956), of a tunnel created in an ice fall in one of the Norwegian glaciers. When a tunnel is dug into an ordinary glacier to depths in which the dominant stress is primarily the hydrostatic pressure P (equal to the ice overburden), the closure rate according to the theory of Nye (1953) is equal to CP^n, where n is the power-law exponent and C is a constant. However, large stress components other than hydrostatic pressure exist in an ice fall. Glen estimated that a compressive stress $\bar{\sigma}$ of the order of 300 kPa existed in the vicinity of his tunnel. This stress is larger in magnitude than hydrostatic pressure. In such a situation it is reasonable to expect, from the theory of creep under multi-stress components of this section, that the closure rate will be of the order of $C\bar{\sigma}^{(n-1)}P$. Thus the closure rate is linearized with respect to P and the closure rate is increased. In Figure 3 are shown Glen's data, which are treated as if $\bar{\sigma}$ were equal to zero. It can be seen that the closure rate is indeed approximately proportional to P, and is considerably larger than the usual closure rate that is proportional to P^3.

In glacier and ice sheet flow problems similar multi-stress component effects occur. Moreover, different ice fabrics develop under different stress components. In a glacier there exist at least two stress components: a shear stress that acts across any plane that is parallel to the bed of the glacier, and a longitudinal compressive or tensile stress component that acts along the

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length of the glacier. They both influence the flow that causes a borehole to tilt, making it more difficult to interpret tilt measurements.

Effect of Hydrostatic Pressure

The effect that hydrostatic pressure has on the creep rate is not well established at the present time. The melting temperature of ice changes in an anomalous manner with pressure. It decreases, rather than increases, with pressure at a rate of -7.4×10^{-8}°C Pa^{-1}. Thus it is reasonable to expect that increasing the pressure at a constant test temperature will increase the creep rate because the melting temperature is brought closer to the test temperature. The first determinations of the activation volume gave $V = -2 \times 10^{-5}$ m^3 mole^{-1} $= -3.3 \times 10^{-29}$ m^3 (review W). These two experiments are preliminary (see review W), but they do show that the creep rate is increased by an amount expected from the decrease in the melting temperature. However, Higashi & Shoji (1979) find in a constant strain rate experiment on Antarctic ice that the maximum stress of the monotonically increasing stress-strain curve increases with the hydrostatic pressure. This result implies that hydrostatic pressure decreases the creep rate. Higashi & Shoji attribute the hardening increase with pressure to the fact that air bubbles in the ice are reduced in volume and cleavage cracks are closed by the higher pressure.

Very recent experiments by Jones & Chew (1982) on polycrystalline ice tested at -9.6°C under a compressive stress of 0.525 MPa gave the surprising result that the creep rate *decreased* from 8.3×10^{-9} s^{-1} to 7×10^{-9} s^{-1} when the superimposed hydrostatic pressure was increased from 0.1 to 15 MPa. The creep rate went through a minimum at pressures between 15 and 30 MPa. Above 30 MPa the creep rate *increased* with increase in pressure, and reached a value of 17×10^{-9} s^{-1} at a pressure of 60 MPa. The activation volume changed from $+3.2 \times 10^{-5}$ m^3 mole^{-1} $=$ 5.3×10^{-29} m^3 to -5.5×10^{-5} m^3 mole^{-1} $= -9.2 \times 10^{-29}$ m^3 as the pressure was increased. Jones points out that these results imply that several processes are controlling the creep rate. (We have already seen that different processes probably control the rate of easy glide and hard glide. Perhaps Jones' results involve both these mechanisms.) Recent results of Durham et al (1982) indicate that the activation volume is negative.

The effect of hydrostatic pressure on grain growth has been studied by Azuma & Higashi (1982). The rate of growth increases with pressure as well as temperature. The activation energy is 77 kJ mole^{-1} and the activation volume is -6.68×10^{-5} m^3 mole^{-1} $= -11 \times 10^{-29}$ m^3. Paterson (1981, p. 18) has determined an activation energy of grain growth in polar firn of only 42.3 kJ mole^{-1}.

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VELOCITY OF INDIVIDUAL DISLOCATIONS

The velocity of individual dislocations in ice can be measured directly. In Figure 6 are plotted measured velocities of dislocations on the basal plane of pure ice versus applied basal shear stress. The velocity is proportional to the stress. The activation energy for motion determined by varying the temperature is approximately the same as the creep activation energy. [Mai et al (1978) have found that dislocations move about 50% faster in ice that is doped with HF.]

A partly phenomenological, partly theoretical creep rate can be found by using the Figure 6 data in creep equation (1) and by using (3) to estimate the steady-state dislocation density. In (1) and (3) use $\alpha = \beta = 1$, $b = 4.5 \times 10^{-10}$ m, $\mu = 3 \times 10^9$ Pa, and assume that the Figure 6 data are reasonably accounted for if $v = 6 \times 10^{-7}$ m s^{-1} at a shear stress of 0.1 MPa. The creep rate versus stress plot calculated this way is shown in Figure 3 (where shear creep rate and shear stress have been converted to the longitudinal creep rate and stress appropriate for uniaxial

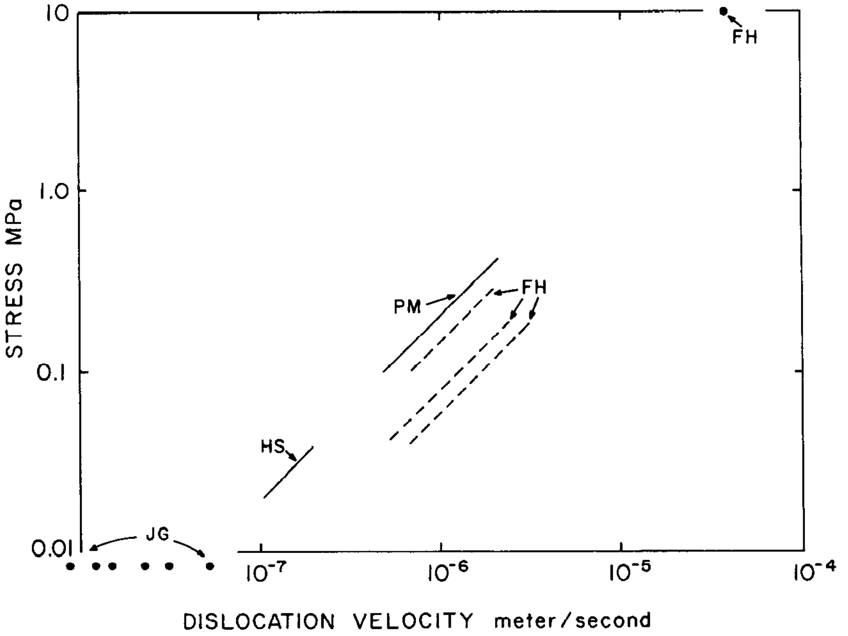

Figure 6 Dislocation velocity (normalized to $-10°$C using $Q = 60$ kJ/mole) versus stress. Data from the following sources. FH: Fukuda & Higashi (1973); JG: Jones & Gilra (1973); HS: Higashi & Sake (given in FH); PM: Perez et al (1978) and Mai et al (1978).

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tension and compression). This calculated creep rate can only be accurate to about a factor of about 5 because of the rough nature of the analysis. It appears, therefore, that easy glide can be accounted for since the easy-glide curve and the calculated curve lie within a factor of 2 to 3 of each other in Figure 3.

A complete theory also would account for the velocity-stress relationship of Figure 6. Mechanisms that give a linear velocity-stress relationship involving stress-induced order of protons in the stress field of a dislocation and bond-breaking proton rearrangements and/or defect reorientations in the core regions of dislocations have been proposed. The stress-induced order mechanism predicts a dislocation velocity of 5×10^{-7} m s^{-1} at $-10°C$ and at a stress of 0.1 MPa (see review W). This value agrees quite well with the data of Figure 6. (However, in review W it is pointed out that this mechanism probably can be ruled out if the creep rate indeed increases with pressure.)

The core region mechanism, which has been analyzed repeatedly (Glen 1968, Perez et al 1975, Whitworth et al 1976, Frost et al 1976, Forouhi & Bloomer 1978, Whitworth 1980, 1982, Goodman et al 1981) initially appeared to give a basal dislocation glide velocity much smaller than observed. However, more recent developments of the theory make it appear likely that dislocations controlled by this mechanism do move with about the right velocity.

Polycrystalline ice and ice in a hard orientation creep more slowly by a factor of about 500 than ice of a soft orientation. If the second glide-controlled mechanism does apply to glide on nonbasal planes with predicted dislocation velocities as low as found in the original theories, it might account for the lower creep rates in the harder ice.

Dislocation climb, however, cannot be ruled out as the rate-controlling process of the harder ice. The climb velocity of an isolated edge dislocation is approximately equal to (Weertman 1975)

$$v \simeq (D/2b)(\sigma\Omega/kT), \tag{15}$$

where $\Omega = 3.2 \times 10^{-29}$ m^3 is the molecular volume for ice, $k = 1.38 \times 10^{-23}$ J K^{-1} = Boltzmann's constant, and $D = D_0 \exp(-Q/kT)$ is the self-diffusion coefficient for ice. Here $D_0 \simeq 1.5 \times 10^{-3}$ m^2 s^{-1} and $Q =$ 60 kJ $mole^{-1}$ for both hydrogen and oxygen diffusion (see review W). At $T = 263$ K ($-10°C$) and $\sigma = 0.1$ MPa the climb velocity v is equal to $v = 2.0 \times 10^{-9}$ m s^{-1}. This velocity is a factor of 300 smaller than the experimentally determined glide velocity of 6×10^{-7} m s^{-1} given above, a factor of similar magnitude as that between the creep rate of the soft ice and the hard ice. Therefore the climb mechanism can account quantitatively for the creep of the harder ice. The deceleration in the creep rate of the creep

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curve and, in the constant strain rate test, the monotonic increase of the stress-strain curve of the hard ice are explained if the glide motion is faster at the same stress level than the climb motion. Thus the exercise given earlier is partly solved. But now another problem remains to be solved: If the climb velocity is so small compared with the glide velocities of Figure 6, why doesn't climb control the creep rate of easy-glide single crystals and cause the creep rate of such crystals to be about the same as that of polycrystalline ice? [Duval & Ashby (1982) have concluded that glide-controlled creep is not important for polycrystalline ice and that recovery dislocation processes, which I assume involve dislocation climb, may be.]

Discussions of other creep mechanisms proposed for ice are given by Landgon (1973), Goodman et al (1977, 1981), Gilra (1974), Perez et al (1978), and in reviews G and W.

CREEP OF ICE OTHER THAN ICE Ih

Only very limited creep results on ice in a crystalline form other than ice Ih have been obtained up to now. Poirier et al (1981) measured an effective viscosity of ice VI at room temperature at pressures in the range of 1.1–1.2 GPa in a sapphire anvil press. They found a viscosity of 10^{14} poise (10^{13} Pa s). The shear stress where this effective viscosity is determined is of the order of 90 MPa and the creep rate in shear of the order of $3 \times 10^{-6}\ s^{-1}$. Thus this ice is considerably harder than ice Ih. If a third power law is assumed to extrapolate the creep rate to stresses of the order of 1 to 10 MPa (10 to 100 bars), the effective viscosity is increased to 10^{17} to 10^{20} poise.

Poirier (1982) has briefly described unpublished preliminary results of Echelmeyer & Kamb on the effective viscosity of ice II and ice III. Ice II is somewhat more viscous than ice Ih at a given temperature (but different pressure), but ice III is three orders of magnitude less viscous than ordinary ice. Durham et al (1982, private conversation) have studied the ductile and the brittle deformation of ice at pressures up to 350 MPa and temperatures between 77 and 195 K. The brittle strength of a higher density ice phase was found to be 156 MPa at 158 K and 350 MPa. Lack of X-ray data prevented identification of the phase, which probably is ice IX but could be ice II.

CONCLUSION

The quasi-steady-state creep rate of coarse grain and single crystal ice at moderate stress levels and at relatively large strains is best described by a power-law creep equation with a power exponent equal to three. The activation energy of creep, 60 kJ/mole, is the same as that of self-diffusion of hydrogen and oxygen. The creep rate of single crystals oriented for easy-

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glide, basal slip can be accounted for with experimentally measured values of the glide velocity of individual basal dislocations. The creep mechanism of polycrystalline ice and ice single crystals in a hard orientation in which basal slip is not possible is not certain. Although a dislocation climb mechanism can account for the observed creep rates for this harder ice, it is not possible to conclude that this mechanism actually is rate controlling.

For purposes of ice modeling the best constants to use in the equivalent of a uniaxial tension or compression test appear to be a creep rate of $10^{-10}\ s^{-1}$ under a stress of 0.1 MPa and a temperature of $-10°C$. To obtain creep rates at other stress levels, a power exponent of $n = 3$ can be used. To obtain the creep rates at other temperatures, the activation energy to use is 60 kJ $mole^{-1}$ for single crystal ice and polycrystalline ice below $-10°C$. For polycrystalline ice above $-10°C$ the activation energy to use is 134 kJ $mole^{-1}$. These values of creep rate, power exponent, and activation energy are almost identical to those recommended by Paterson (1981, p. 39, and private communication) when his constants are converted into those appropriate to the uniaxial stress condition. In polycrystalline ice with the *c*-axis strongly aligned to the vertical, the creep rate in shear parallel to the glacier bed ought to be increased by a factor of four. The effect of hydrostatic pressure in any ice sheet or glacier on the creep rate should be rather small, and except for the thickest ice sheets can ordinarily be ignored.

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