# A study of trajectories to the Neptune system using gravity assists 

C.R.H. Solórzano ${ }^{\text {a }}$, A.A. Sukhanov ${ }^{\text {b }}$, A.F.B.A. Prado ${ }^{\text {a,* }}$<br>${ }^{a}$ National Institute for Space Research (INPE), São José dos Campos, SP 12227-010, Brazil<br>${ }^{\mathrm{b}}$ Space Research Institute (IKI), Russian Academy of Sciences 84/32, Profsoyuznaya St., 117810 Moscow, Russia

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#### Abstract

At the present time, the search for the knowledge of our Solar System continues effective. NASA's Solar System Exploration theme listed a Neptune mission as one of its top priorities for the mid-term (2008-2013). From the technical point of view, gravity assist is a proven technique in interplanetary exploration, as exemplified by the missions Voyager, Galileo, and Cassini. Here, a mission to Neptune for the mid-term (2008-2020) is proposed, with the goal of studying several schemes for the mission. A direct transfer to Neptune is considered and also Venus, Earth, Jupiter, and Saturn gravity assists are used for the transfer to Neptune, which represent new contributions for a possible real mission. We show several schemes, including or not the braking maneuver near Neptune, in order to find a good compromise between the $\Delta V$ and the time of flight to Neptune. After that, a study is made to take advantage of an asteroid flyby opportunity, when the spacecraft passes by the main asteroid belt. Results for a mission that makes an asteroid flyby with the asteroid 1931 TD3 is shown.


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## 1. Introduction

Among the first researches that approached the problems of interplanetary missions, we have the study made by Hollister and Prussing (1966), which considered a Mars transfer through Venus, analyzing the advantages of an impulsive maneuver during the close approach with Venus. One of the pioneering works in its time, for the vision of trips to our solar system using the concepts of the grav-ity-assist maneuver, was the work developed by Flandro (1966), which planned a mission to the exterior solar system using the energy of the gravitational field of Jupiter.

Later, D'Amario et al. (1981) developed several procedures with the goal of minimizing the total impulse $\Delta V$ for multiple-flyby trajectories with constraints on the flyby parameters and in the times of the maneuvers. This procedure successfully optimized the Galileo satellite tour, which

[^0]contained up to eleven flybys. After that, D'Amario et al. (1982) modified the procedure for minimizing the total impulsive $\Delta V$ for application to interplanetary trajectories. Examples of the application of this new method are given for several types of Galileo interplanetary trajectory options. Then, Carvell (1986), studied the Ulysses mission that used a close approach with Jupiter to change its orbital plane to observe the poles of the Sun.

Longuski and Williams (1991) analyzed multiple encounter trajectories to the far outer planets. The most significant result is the last four-planet grand tour, with encounters with Jupiter, Uranus, Neptune, and Pluto. Other missions were designed, including Jupiter and only one or two of the other planets, but they have short flight times. Then, Longuski et al. (1991) considered a new approach to planetary mission design. This new design tool is applied to the problems of finding multiple encounter trajectories to the outer planets and Venus gravity-assist trajectories to Mars.

Striepe and Braun (1991) analyzed missions to Mars using the technique of maneuvers assisted by the gravity of Venus. This maneuver provides a non-propulsive change
in the heliocentric energy of the spacecraft that can reduce the amount of propellant necessary to complete the interplanetary mission and/or to reduce the duration of some missions. For certain positions of the planets, it incorporates a propulsive maneuver.

Swenson (1992) considered a mission to Neptune using a gravity assist maneuver with the Earth and Jupiter, besides considering multiple gravity assists with Venus (VVEJGA, which means Venus-Venus-Earth-Jupiter-gravity-assist). It also considered a combination with propulsive maneuvers.

Patel et al. (1995) investigated multiple-encounter missions to the outer planets which can also include a passage by Pluto. A particularly important result of this analysis is the discovery of a three-planet trip opportunity that includes Uranus, Neptune, and Pluto.

Peralta and Flanagan (1995) planned the interplanetary trajectories of the Cassini mission. The trajectory with multiple gravity assists with Venus-Venus-Earth-Jupiter supplies the energy required to reach Saturn. In this way, the geometry of the trajectory VVEJGA provides one technique to double the gravitational assistance with Venus.

Sims et al. (1997) analyzed several trajectories to Pluto using a gravity-assist maneuver with Jupiter. They also analyzed a gravity-assist maneuver with Mars, in conjunction with three maneuvers assisted by Venus.

Sukhanov (1999) studied a mission to the Sun, by means of gravity assists with the interior planets. It considers maneuvers with the Earth, Mars, and Venus. There are advantages in the cost, when compared to the gravity-assist maneuver with Jupiter, making it possible to use multiple maneuvers with the planet Mercury.

Exploration of small bodies in the solar system, in particular with asteroids, is among the most promising lines of space trajectory studies. In this way, Galileo was the first space mission to make a close approach to the asteroids (Belton and Delamere, 1992) denominated Gaspra (951) and Ida (243). It was the beginning of a refined study of the asteroids "in situ," exemplified by the satellite Near, which went to Eros (433).

In the present paper, a mission to Neptune is proposed for the time period 2008-2020. The main goal is to study several schemes for the mission. A direct transfer is considered, as well as trajectories that perform gravity assists with the planets Venus, Earth, Jupiter, and Saturn. The results represent new contributions for a possible real mission. Schemes including or not the braking maneuver near Neptune are considered, in order to find a good compromise between the $\Delta V$ and the time of flight to Neptune. Then, a study is made to consider an asteroid flyby, when the spacecraft passes by the main asteroid belt. Simulations are made considering the asteroid 1931 TD3.

## 2. Method for the development of the mission

The trajectory of the spacecraft is represented by a series of segments of undisturbed Keplerian motion around the
gravispheres of relevant celestial bodies while, on the boundaries of these segments, the trajectories goes from the gravisphere into the heliosphere and vice-versa. Ordinarily, this planetary maneuver provides a non-propulsive change in the spacecraft's heliocentric energy which can reduce the amount of propellant required to complete an interplanetary mission. The heliocentric energy may be increased or decreased, depending on the geometric details of the encounters (turn of velocity vector over the sphere of influence of the planet).

The planetary orbits are elliptic and non-coplanar, and take into account the phasing of the planetary motion along the orbits for our analysis. The analysis is made for specific dates of the interplanetary flights (or their intervals) with estimates of minimum energy expenditure for the mission. Segments of heliocentric motion from the Earth to the flyby planet and from the flyby planet to the destination planet are constructed. These segments of the interplanetary trajectory are joined based on the incoming and outgoing excess velocity vectors to the flyby planet. Since the "patched conics" approximation is used, the incoming and outgoing excess velocities with respect to the planet that is in use for the swing-by are equals in the non-propelled swing-by and, when the impulse is applied, they differ by the amount of the impulse applied. For the case of multiple flybys, a similar construction is made for the subsequent segments of the trajectory. With this information, the optimal trajectory is sought based on the criterion of minimum total characteristic velocity $(\Delta V)$. After using the model described above, the optimization problem become a parametric optimization that can be easily solved.

For the schemes that considered the braking near Neptune, the hyperbolic excess velocity at Neptune contributes to the total $\Delta V$. In this case, the braking impulse is applied when the spacecraft reaches a distance that corresponded to $5 \%$ of radii of Neptune, independent of the keplerian elements of the incoming orbit. Thus, the impulsive maneuver changes the hyperbolic orbit to a parabolic orbit near Neptune.

Earth and Venus are the inner planets that have a gravity field large enough to be used. Jupiter and Saturn show optimum launch opportunities for flights to Neptune using the energy gained during the close approach. However, to approach Neptune closely, the spacecraft should have low excess velocity to reduce the cost of the braking maneuver. The optimal launch date in the time interval $2008-2020$ is considered. The following transfer schemes are analyzed: Direct Earth to Neptune (EN) transfer, Earth-Jupiter-Neptune (EJN) transfer, Earth-SaturnNeptune (ESN) transfer, Earth-Jupiter-Saturn-Neptune (EJSN) transfer, Earth-Venus-Earth-Jupiter-Neptune (EVEJN) transfer and Earth-Venus-Earth-Jupiter-SaturnNeptune (EVEJSN) transfer.

## 3. The mission options

Considering the requirement of a good compromise between the characteristic velocity ( $\Delta V$ ) and the time of
flight, we analyzed several options of transfers in the time interval 2008-2020. In this interval, Uranus do not have good positions to supply an improvement in the fuel consumption through a gravity-assist maneuver and Mars do not have mass large enough to be used in this mission. Fig. 1 shows the $\Delta V$ for several schemes, not including the braking maneuver, as a function of the time of flight. The time of flight, in years, is represented in the horizontal axis and the total fuel consumption is represented in the vertical axis in terms of the $\Delta V$, in $\mathrm{km} / \mathrm{s}$. The letters identifies the scheme, as described above, and the year is also shown. A figure like this can be used to choose the best scheme, taking into account the fuel consumption and the time of flight.

Table 1 shows that the minimum total $\Delta V$ is $6.506 \mathrm{~km} / \mathrm{s}$ for the EJN scheme and that the total flight duration is 12 years, with a flyby altitude of $0.2 \times 10^{3} \mathrm{~km}$ (Earth) and $1.2 \times 10^{3} \mathrm{~km}$ (Neptune). However, the excess velocity near Neptune is $11.728 \mathrm{~km} / \mathrm{s}$. Another scheme with low $\Delta V$ (minimum total fuel consumption) is EVEJN, however the excess velocity near Neptune is higher than in the EJN option.

Figs. 2-4 show the planetary configuration for the transfer schemes EJN, EVEJN, and EVEJSN, projected on the plane of the ecliptic. The letters represent the planets: E is the Earth, V is Venus, $\mathbf{J}$ is Jupiter, $\mathbf{S}$ is Saturn and N is Neptune. Considering a time of flight of 12 years, the spacecraft flyby Jupiter with an altitude of $4.218 \times 10^{5} \mathrm{~km}$, for the EJN scheme. Other options, as the ESN, shows that the transfer angle Earth-Saturn decreases and the Saturn-Neptune angle is quasi-constant. In this case, the flyby altitude at Saturn is $9.670 \times 10^{4} \mathrm{~km}$.


Fig. 1. Total $\Delta V(\mathrm{~km} / \mathrm{s})$ vs. time of flight (years) for several transfers, not including the braking maneuvers. The following transfer schemes are analyzed: Direct Earth to Neptune (EN) transfer, Earth-Jupiter-Neptune (EJN) transfer, Earth-Saturn-Neptune (ESN) transfer, Earth-Jupiter-Saturn-Neptune (EJSN) transfer, Earth-Venus-Earth-Jupiter-Neptune (EVEJN) transfer, and Earth-Venus-Earth-Jupiter-Saturn-Neptune (EVEJSN) transfer.

Table 1
Optimal transfer schemes for a 12 years mission, showing the launch date, the excess velocity near Neptune $(\mathrm{km} / \mathrm{s})$ and the minimum total $\Delta V(\mathrm{~km} / \mathrm{s})$

| Transfer <br> scheme | Launch <br> date | Excess velocity <br> $V_{\text {inf }}(\mathrm{km} / \mathrm{s})$ | Minimum total <br> $\Delta V(\mathrm{~km} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- |
| EN | 13-Apr-2012 | 9.436 | 8.992 |
| EJN | 14-Jan-2018 | 11.728 | 6.506 |
| ESN | 13-Feb-2016 | 12.955 | 7.775 |
| EJSN | 18-Nov-2015 | 15.757 | 6.719 |
| EVEJN | 24-Aug-2016 | 14.578 | 6.646 |
| EVEJSN | 09-Jun-2015 | 17.275 | 7.206 |



Fig. 2. Planetary configuration and transfer trajectory for a 2018 Earth-Jupiter-Neptune transfer. The letters represent the planets: E is the Earth, J is Jupiter, and N is Neptune.

For the EVEJN and EVEJSN schemes, the transfer angle Earth-Venus decreases, but the transfer angle Venus-Earth is high, however the other transfer angles are quasi-constant, due to the long periods of the planets, when compared to the transfer times. The spacecraft flyby Venus with an altitude of $0.3 \times 10^{3} \mathrm{~km}$, for the EVEJN scheme, and $0.4 \times 10^{3} \mathrm{~km}$, for EVEJSN scheme. Tables 2 and 3 show some schemes considering the time of flight and the total $\Delta V$. Table 4 shows the optimal launch date for several transfers. The minimum total $\Delta V$ is $5.441 \mathrm{~km} / \mathrm{s}$ for the EVEJSN scheme and the total flight duration is 23.69 years. The excess velocity near Neptune is $5.083 \mathrm{~km} / \mathrm{s}$, however the EVEJN scheme has a smaller excess velocity near Neptune ( $3.748 \mathrm{~km} / \mathrm{s}$ ) for the optimal transfer time of 29.95 years. Fig. 1 shows that the Direct Earth to Neptune transfer does not allow a lower value for the energy requirements for such flight. Looking at that, the schemes with gravity-assist has smaller fuel consumptions.

Fig. 5 shows several transfer schemes for transfers from Earth to Neptune. It has the same description made for Fig. 1, but, in this figure, only the most economical schemes are shown. Looking the curves of minimum total $\Delta V$ as a


Fig. 3. Planetary configuration and transfer trajectory for a 2016 Earth-Venus-Earth-Jupiter-Neptune transfer. The letters represent the planets: E is the Earth, V is Venus, J is Jupiter, and N is Neptune.
function of the transfer time, the EJN, EJSN and EVEJN schemes are the most acceptable if the transfer duration is limited by the time of 12 years. The EVEJSN scheme has a minimum total $\Delta V$ similar to the values for the EJN, EJSN, and EVEJN schemes, for transfer times in the interval 13-14 years. For transfer times above 14 years, the EVEJSN scheme is optimal, in terms of minimum total $\Delta V$. Fig. 6 shows the excess velocity near Neptune, in $\mathrm{km} / \mathrm{s}$. This is an important parameter if the goal of the mission is to stay around Neptune and a capture maneuver is required. The EVEJSN scheme is optimal in terms of minimum total $\Delta V$, but the excess velocity near Neptune ( $V_{\text {inf }}$ ) is very high. The EJN and EVEJN schemes are more efficient in terms of having a low $V_{\mathrm{inf}}$ and minimum $\Delta V$. This figure can help the mission designer to choose the best trajectory, taking into account both parameters.

Fig. 7 shows the minimum total $\Delta V$ as a function of the optimal launch date for several transfer schemes in the time interval 2008-2020. Of course, the launch dates for each of the schemes considered are discrete (to be more correct, the launch is possible during rather short launch windows). In
this way, the curves shown in Figs. 7 and 9 just formally approximate these dates. This figure shows a detailed view of the cost of each type of transfer, as functions of the launch date.

Considering an initial Jupiter flyby, it is possible to see that, when the spacecraft has a flyby altitude at Jupiter of $4.218 \times 10^{5} \mathrm{~km}$ (EJN scheme and time of flight of 12 years), the launch $\Delta V$ decreases, but the excess velocity in Jupiter increases. For an Earth-Neptune direct transfer it is shown that, when the launch $\Delta V$ decreases, the $V_{\text {inf }}$ at Neptune also decreases.

The launch $\Delta V$ for the ESN option also decreases, as well as the transfer angle Earth-Saturn and the SaturnNeptune angle is quasi-constant. For the EVEJN and EVEJSN schemes, the transfer angle Earth-Venus decreases, however the other transfer angles are quasi-constant, due to the long periods of the planets, when compared with the transfer times, but the transfer angle of Venus-Earth is high.

Fig. 8 shows the total $\Delta V$ for the transfers, this time considering the braking maneuver near Neptune, as a


Fig. 4. Planetary configuration and transfer trajectory for a 2015 Earth-Venus-Earth-Jupiter-Saturn-Neptune transfer. The letters represent the planets: E is the Earth, V is Venus, J is Jupiter, S is Saturn, and $\mathbf{N}$ is Neptune.

Table 2
$V_{\text {inf }}$ and total $\Delta V$ for the Earth-Saturn-Neptune scheme without braking

| Time of flight <br> (years) | Launch <br> date | Hyperbolic excess <br> velocity $V_{\text {inf }}(\mathrm{km} / \mathrm{s})$ | Minimum total <br> $\Delta V(\mathrm{~km} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- |
| 12 | 13-Feb-2017 | 12.955 | 7.772 |
| 13 | $13-$ Feb-2017 | 11.580 | 7.648 |
| 15 | $13-$ Feb-2017 | 9.353 | 7.530 |
| 18 | 17 -Feb-2017 | 7.040 | 7.515 |

Optimal launch dates for several transfers.
function of the time of flight. It is visible that, in the first 14 years of flight, the EVEJSN scheme has high fuel consumption. However, for larger times of transfer, like in the interval $17-18$ years, this option shows a better performance when the fuel consumption is considered. Besides, Fig. 8 shows that the EJN option has better results when compared to the EVEJN option for times of flight in the interval 12-18 years. The EJN option supplied the best values of $\Delta V$ for a time of flight close to 17 years, and the option EN shows higher values for

Table 3
$V_{\text {inf }}$ and total $\Delta V$ for the Earth-Venus-Earth-Jupiter-Saturn-Neptune scheme without braking

| Time of flight <br> (years) | Launch <br> date | Hyperbolic excess <br> velocity $V_{\mathrm{inf}}(\mathrm{km} / \mathrm{s})$ | Minimum total <br> $\Delta V(\mathrm{~km} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- |
| 12 | 09-June-2015 | 17.275 | 7.206 |
| 13 | 09-June-2015 | 15.153 | 6.837 |
| 17 | 05-June-2015 | 9.614 | 5.432 |
| 18 | 02-June-2015 | 8.646 | 5.428 |

the minimum $\Delta \mathrm{V}$, when compared with the other options of transfers.

The EJN option reveals to be excellent, due to the fact that, for a time of flight of 12 years, it has a minimum $\Delta V$ of $9.298 \mathrm{~km} / \mathrm{s}$ and keeps a comparative excellent behavior, with respect to the other options, until a time of flight of 17 years. However, there is a region with multiple points of intersection of all the options. This happens when the time of flight is next to 14 years. For a time of flight larger than 14 years, the EN, ESN, and EJSN options have high

Table 4
Optimal launch date for several transfer schemes without braking

| Transfer <br> scheme | Optimal <br> launch date | Minimum total $\Delta V$ <br> $(\mathrm{~km} / \mathrm{s})$ | Optimal transfer <br> time (years) |
| :--- | :--- | :--- | :--- |
| EN | 09-Apr-2009 | 8.691 | 19.59 |
| EJN | 13-Jan-2018 | 6.367 | 19.78 |
| ESN | 17-Jan-2014 | 7.273 | 19.72 |
| EJSN | 26-Nov-2015 | 6.428 | 25.70 |
| EVEJN | 28-May-2013 | 5.642 | 29.95 |
| EVEJSN | 30-May-2015 | 5.441 | 23.69 |



Fig. 5. Total $\Delta V(\mathrm{~km} / \mathrm{s})$ vs. time of flight (years) for the best of each transfer shown in Fig. 1. The following transfer schemes are analyzed: Direct Earth to Neptune (EN) transfer, Earth-Jupiter-Neptune (EJN) transfer, Earth-Saturn-Neptune (ESN) transfer, Earth-Jupiter-SaturnNeptune (EJSN) transfer, Earth-Venus-Earth-Jupiter-Neptune (EVEJN) transfer, and Earth-Venus-Earth-Jupiter-Saturn-Neptune (EVEJSN) transfer.
fuel consumption, when compared to the EVEJN, EJN, and EVEJSN options. The EVEJN option has also low fuel consumption until a time of flight of 15.5 years. The time of flight of ESN and EVEJSN schemes are shown in Tables 5 and 6 , where several values of $V_{\mathrm{inf}}$ and $\Delta V$ are shown.

The EVEJN option has low fuel consumption until a time of flight of 15.5 years, but the EVEJSN option results with lower consumption. The options shown were simulated considering dates of launching in 2012 for the Earth to Neptune transfer, in 2018 for Earth-Jupiter-Neptune transfer, 2016 for Earth-Saturn-Neptune transfer, 2015 for the Earth-Jupiter-Saturn-Neptune transfer, 2016 for the Earth-Venus-Earth-Jupiter-Neptune transfer, and 2015 for the Earth-Venus-Earth-Jupiter-Saturn-Neptune transfer. Fig. 9 supplies information of optimal launch date for maneuvers including the braking cost near Neptune in the time interval 2008-2020. The EVEJSN option has the best value of $\Delta V$, thus the minimum value is $5.899 \mathrm{~km} / \mathrm{s}$ for a time of flight of 31.20 years. Another option with low value of $\Delta V$ is EVEJN.

We studied two important parameters, the total $\Delta V$ and the excess velocity $V_{\text {inf }}$ near Neptune. They are considered


Fig. 6. $V_{\text {inf }}$ near Neptune ( $\mathrm{km} / \mathrm{s}$ ) vs. time of flight (years) for several types of transfers. The following transfer schemes are analyzed: Direct Earth to Neptune (EN) transfer, Earth-Jupiter-Neptune (EJN) transfer, Earth-Saturn-Neptune (ESN) transfer, Earth-Jupiter-Saturn-Neptune (EJSN) transfer, Earth-Venus-Earth-Jupiter-Neptune (EVEJN) transfer, and Earth-Venus-Earth-Jupiter-Saturn-Neptune (EVEJSN) transfer.


Fig. 7. Optimal launch date for several transfer schemes not including the braking maneuvers. The following transfer schemes are analyzed: Direct Earth to Neptune (EN) transfer, Earth-Jupiter-Neptune (EJN) transfer, Earth-Saturn-Neptune (ESN) transfer, Earth-Jupiter-Saturn-Neptune (EJSN) transfer, Earth-Venus-Earth-Jupiter-Neptune (EVEJN) transfer, and Earth-Venus-Earth-Jupiter-Saturn-Neptune (EVEJSN) transfer.
as a function of the date of launching and the time of flight. These two parameters determine, respectively, the fuel consumption to leave a Low Earth Orbit (LEO), around 500 km altitude and the braking maneuver near Neptune, so the $\Delta V$ was considered the most important parameter. The EJN option without the cost of braking has a minimum $\Delta V$ for a transfer whose duration is less than 14 years. This option also determines low values for $V_{\text {inf }}$. For larger times of transfer, option EVEJSN is excellent in terms of


Fig. 8. Total $\Delta V$ vs. time of flight for maneuvers including the braking near Neptune. The following transfer schemes are analyzed: Direct Earth to Neptune (EN) transfer, Earth-Jupiter-Neptune (EJN) transfer, Earth-Saturn-Neptune (ESN) transfer, Earth-Jupiter-Saturn-Neptune (EJSN) transfer, Earth-Venus-Earth-Jupiter-Neptune (EVEJN) transfer, and Earth-Venus-Earth-Jupiter-Saturn-Neptune (EVEJSN) transfer.

Table 5
$V_{\mathrm{inf}}$ and total $\Delta V$ for the Earth-Neptune scheme with braking

| Time of flight <br> (years) | Launch <br> date | Hyperbolic excess <br> velocity $V_{\mathrm{inf}}(\mathrm{km} / \mathrm{s})$ | Minimum total <br> $\Delta V(\mathrm{~km} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- |
| 12 | 14-Apr-2012 | 9.436 | 10.837 |
| 14 | 14-Apr-2012 | 7.735 | 10.051 |
| 15 | 15-Apr-2012 | 7.076 | 9.805 |
| 18 | 18-Apr-2012 | 5.658 | 9.418 |

the minimum $\Delta V$; however $V_{\text {inf }}$ is high in this option. To find the optimal dates of launching, the options EJN, EVEJN, and EVEJSN are more acceptable. If the duration of the transfer is limited to 14 years or less, option EJN is preferable in all aspects. The EVEJSN option is preferable for times of transfer larger than 14 years.

The transfer scheme including the braking maneuver has high energy requirements. The optimal launch date for each of the transfer options had been kept with the goal of making a better comparison. Thus, we observe that the EVEJSN option is still preferential for times of flight near 18 years (Figs. 5 and 8). The EJN option including the braking maneuver has an excellent behavior in terms of fuel consumption when compared to the other options for times of flight between 12 and 17 years. However, in

Table 6

| $V_{\text {inf }}$ and total $\Delta V$ for the Earth-Jupiter-Neptune scheme with braking |  |  |  |
| :--- | :--- | :--- | :--- |
| Time of flight | Launch | Hyperbolic excess <br> (years) |  |
| 12 | date | velocity $V_{\text {inf }}(\mathrm{km} / \mathrm{s})$ | $\Delta V(\mathrm{~km} / \mathrm{s})$ |
| 14 | 14-Jan-2018 | 11.720 | 9.298 |
| 15 | 14-Jan-2018 | 9.323 | 8.215 |
| 18 | 14-Jan-2018 | 8.391 | 7.859 |



Fig. 9. Optimal launch date for several transfer schemes for trajectories including the braking maneuvers near Neptune. The following transfer schemes are analyzed: Direct Earth to Neptune (EN) transfer, Earth-Jupiter-Neptune (EJN) transfer, Earth-Saturn-Neptune (ESN) transfer, Earth-Jupiter-Saturn-Neptune (EJSN) transfer, Earth-Venus-Earth-Jupiter-Neptune (EVEJN) transfer, and Earth-Venus-Earth-Jupiter-Saturn-Neptune (EVEJSN) transfer.
the simulations made without including the braking cost (Fig. 5), we observe that the variations are small in that interval of time of flights. For a time of flight close to 18 years, the EVEJSN, EJN, EVEJN, and EJSN are preferred.

The main characteristic of the optimal launch date is the fact that, in the case of transfers including the braking costs, the transfer time is greater than in the cases not including the braking costs. It is observed that the options that consider the braking costs have larger fuel consumption, however, the optimal launch date requires larger times of transfer. Other schemes are shown in Table 7.

## 4. Flyby to asteroid 1931 TD3

The exploration of our outer solar system can also be improved by taking advantage of asteroid flyby opportunities, when the spacecraft passes through the asteroid belt. To incorporate an asteroid flyby, we first need to optimize a trajectory to Neptune with planetary flybys and then search for asteroids that pass close to this trajectory. Then,

Table 7
Optimal launch date for several transfer schemes with braking

| Transfer <br> scheme | Optimal <br> launch date | Minimum total $\Delta V$ <br> $(\mathrm{~km} / \mathrm{s})$ | Optimal transfer <br> time (years) |
| :--- | :--- | :--- | :--- |
| EN | 29-Apr-2014 | 9.370 | 21 |
| EJN | 15-Jan-2018 | 6.594 | 29.55 |
| ESN | 25-Jan-2015 | 7.877 | 33.81 |
| EJSN | 26-Nov-2015 | 6.648 | 29.06 |
| EVEJN | 28-May-2013 | 5.899 | 31.50 |
| EVEJSN | 01-Jun-2015 | 5.647 | 30.36 |



Fig. 10. Planetary configuration for the flyby of the asteroid 1931 TD3. The letters represent the planets: E is the Earth, V is Venus, J is Jupiter, N is Neptune, and 1931 TD3 is the asteroid.
we finally optimize again the trajectory, including one or more asteroid flybys. With this technique, we considered the flyby of the asteroid 1931 TD3 of the main belt, using the EVEJN scheme. Fig. 10 shows the planetary configuration projected on the plane of the ecliptic. The letters represent the planets: E is the Earth, V is Venus, J is Jupiter, N is Neptune and 1931 TD3 is the asteroid. However, the $\Delta V$ required for the new optimization of the trajectory to reach the asteroid is approximately $0.165 \mathrm{~km} / \mathrm{s}$. For this flyby, $V_{\text {inf }}$ is high, and the use of this flyby for a mission involving a landing on the asteroids is not practical.

## 5. Conclusions

In this paper, two important parameters, the minimum total $\Delta V$ and the excess velocity $V_{\text {inf }}$ near Neptune were obtained as a function of the launch date and flight duration. These two parameters determine the fuel consumption to launch from LEO, midcourse corrections and to brake the spacecraft near Neptune. For schemes not considering the braking maneuver, the EJN scheme provides minimum total $\Delta V$ for transfer durations smaller than 14 years. This scheme also gives relatively low $V_{\text {inf }}$. For longer transfers, the EVEJSN scheme is optimal in terms of minimum total $\Delta V$, however $V_{\mathrm{inf}}$ is high.

All the previous schemes allow a passage near Neptune and, depending on the objectives of the mission, it is possible to make a flyby or to remain in orbit around some of the moons of the planet. In the present case, we need to change the trajectory of the spacecraft to keep it in orbit around Neptune. Then, we apply the braking maneuver in the proximity of Neptune. The EJN scheme provides minimum total $\Delta V$ for transfer durations smaller than 17 years. For longer transfers (17-18 years), the EVEJSN scheme is optimal in terms of minimum total $\Delta V$. The EJN and EVEJSN schemes are most acceptable for longer transfers.

As mentioned in the beginning of this paper, Mars and Uranus are not in advantageous position for the years considered in the present research to be used in the trajectory. This fact is also shown in the work of Longuski and Williams (1991), which shows optimal launches to Neptune between 2005-2007 and 2021-2022, through diverse gravity assists with Jupiter and Uranus. A point of agreement with the mentioned work is the consideration of possible gravity assists with Jupiter and Saturn to arrive in Neptune in the years 2016-2019.

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[^0]:    * Corresponding author. Tel.: +55 12 39456201; fax: +55 1239456226.

    E-mail addresses: renato@dem.inpe.br (C.R.H. Solórzano), sukhanov @iki.rssi.ru (A.A. Sukhanov), prado@dem.inpe.br (A.F.B.A. Prado).

