

Constraining the thickness of Europa's water–ice shell: Insights from tidal dissipation and conductive cooling



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ABSTRACT

The time of crystallization of a 100 km thick ocean on Europa is estimated using a Stefan-style solidification solution. This solution is then extended to estimate the present thickness of the ice shell. It is assumed that the shell is initially in a steady-state conductive regime, and the ocean is taken to be an infinite liquid half space cooling from above. We find that in the absence of tidal heating and without the presence of low-eutectic impurities to serve as anti-freezes, a 100 km thick ocean solidifies in about 64 Myr. Conversely, when considering the present thickness of Europa's ice shell, if tidal heating is included at a global dissipation rate of ~ 1 TW, the shell is found to be, on average, approximately 28 km thick. However, if this dissipative heating is solely restricted to the shell, the local rate of heating may vary significantly due to crustal compositional heterogeneities and it is shown that this process may, in turn, produce thermal maxima in the crust, which could lead to local melting and structural instabilities, perhaps associated with the formation of chaos regions. Our approach is also extended to Ganymede and Callisto in order to estimate the time of solidification of their putative subsurface oceans and the current thicknesses of their ice-I shells.

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1. Introduction

Estimated thicknesses for Europa's ice shell range from a few kilometers or less (Carr et al., 1998; Greenberg et al., 1998, 1999; Williams and Greeley, 1998; Turtle and Pierazzo, 2001; O'Brien et al., 2002) to ≥ 10 km (Ojakangas and Stevenson, 1989; Pappalardo et al., 1998; Rathbun et al., 1998; McKinnon, 1999; Schenk, 2002; Figueredo et al., 2002; Sotin et al., 2002; Schenk, 2002; Hussmann et al., 2002; Tobie et al., 2003). Europa's crustal thickness is central to understanding the formation and development of various surface features such as chaos and lenticulae, and also for regulating the potential for ascent and eruption of cryomagmatic melts. Therefore, placing firmer constraints on its value is vitally important to obtaining a more complete understanding of the various geophysical processes that are currently occurring, or that may have recently occurred, on the icy moon.

Complete solidification of Europa's ocean must be forestalled by a high degree of internal heating due mainly to tidal effects in the

Jovian system. This process, clearly being in a near or quasi steady state and having existed for billions of years, has likely contributed greatly to the ongoing evolution of the ice shell. Judging from the heterogeneous nature of Europa's surface features (Greeley et al., 1998, 2000; Pappalardo et al., 1999), the crust itself is also likely to be highly heterogeneous. Although later we will consider the possible sequence of events leading to Europa's present state of an extensive ocean underlying a relatively thin crust, here we simply use this state as an initial condition.

We therefore assume that Europa's icy shell was formed by the gradual freezing of its ocean from above, and employ a Stefan-style (i.e., Neumann's solution) solidification solution to analyze three slightly different situations: first, in order to set an upper bound on the solidification timescale, an estimate is made of the time of solidification of the subsurface ocean in the absence of any internal heat sources; second, bounds are placed on the thickness of the ice shell assuming a global dissipation rate as a steady heat source; and third, when dissipation is restricted to solely take place within the ice shell, the thermal regime of the shell is investigated as a function of its thickness and the local heating rate, both of which may vary due to compositional heterogeneities in the shell. These results are then employed to discuss possible present day geological processes on Europa.

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Finally, we extend our analyses to investigate the potential for internal oceans in Ganymede and Callisto and the corresponding thicknesses of their icy crusts.

2. The Stefan problem

The change of phase of water from liquid to solid and the resulting release of latent heat involved in the growth of Europa's shell is akin to the process of cooling of a sheet of magma, which has long been studied (e.g., Jaeger, 1965, 1968; Marsh, 1989, 2007). The striking differences, of course, are: (1) the water–ice system solidifies at a specific temperature, whereas silicate magmas have a pronounced liquidus and solidus which are separated by 200° or more, and (2) the ensuing solid in silicate systems is more dense, and thus of smaller volume, than the melt from which it came. Nevertheless, this method of analysis is applicable to investigating the crystallization of Europa's ocean.

Here, Neumann's solution of the Stefan Problem is considered, which describes the cooling and solidification of an infinite half space of liquid where solidification takes place at the solidification front, i.e., the well-defined boundary separating solid and liquid phases, along which the liquid crystallizes to a single, pure compound. The solidification front is thus the plane that moves in response to the addition and subtraction of heat from the system. Assuming an initially ice free, pure water ocean on Europa, the solidification front, $S(t)$, measures the position of the ice–water interface over time, which defines the instantaneous thickness of the ice shell in response to the gradual cooling of the ocean (Fig. 1). The full solution to the Stefan Problem is given in the Appendix A. This solution is obtained by solving the diffusion equation in each medium, ice and water, and subsequently matching the solutions at the interface (i.e., at $S(t)$) through the boundary condition that the upward heat flux out of the ice is balanced by the sum of the heat flow from the underlying water and latent heat released during the progressive freezing of the ocean (see boundary conditions (iv) and (v) in the Appendix A). A well-known result of Neumann's Solution is that the position of the solidification front is given exactly by:

$$S'(t) = 2b\sqrt{F_1} \quad (1)$$

(see (A8) in the Appendix A) where $S'(t) = (S(t)/L)$ is the non-dimensional form of $S(t)$, and $F_1 = \kappa_1 t/L^2$, where κ_1 is the ice thermal diffusivity and L is the ice shell thickness. The constant b contains all the effects of both the latent heat (H) and the contrasting

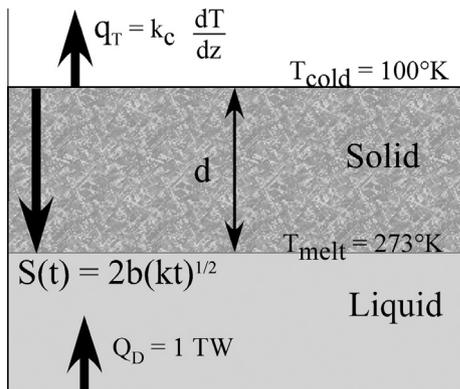


Fig. 1. Geometry for solidification of Europa's subsurface ocean. Here, the ice shell is represented as the solid layer, while the subsurface ocean is represented as the layer of liquid. The upper surface of the crust is assumed to have the constant temperature, $T = 100$ K, while the boundary between the crust and ocean has a constant temperature, $T = 273$ K. Here, q_T represents the conductive heat loss that promotes advancement of the solidification front. Basal heating from tidal dissipation is added to the system at a rate of ~ 1 TW.

thermal diffusivities (κ_1 and κ_2) between ice and water. This equation states that the ice shell grows in direct proportion to the product of the square root of the dimensionless Fourier number ($\kappa_1 t/L^2$) and the constant b , which is always of order unity (i.e., $0 < b < 1$). After applying the appropriate boundary conditions, the transcendental equation for determining the constant b as a function of all the thermal properties and the prevailing temperatures in the system emerges, namely:

$$be^{b^2} \operatorname{erf}(b) = \frac{c_{p1}(T_{\text{melting}} - T_{\text{cold}})}{H\sqrt{\pi}} \quad (2)$$

where c_{p1} is the specific heat at constant pressure of the solid phase (i.e., ice), T_{melting} is the temperature at which the liquid phase melts, and T_{cold} is the surface temperature. The mathematical procedure necessary to obtain this expression is reviewed in detail in the Appendix A.

In the following sections, we will apply (1) and (2) to estimate the timescale for complete crystallization of Europa's ocean and the thickness of the ice shell.

3. Solidification of Europa's ocean

To estimate the time of complete solidification of Europa's ocean, the ocean itself is taken to be an initial liquid half space freezing from the top down, with no internal heating. Since Europa's hydrosphere is thin (~ 100 km) relative to the radius of the moon itself, solidification may be approximated as a flat ice slab overlying a sheet of water. The rate of freezing of the ice shell may then be measured by the temporal progression of the solidification front, $S(t)$ (Fig. 1).

The temporal progression of the solidification front is described above by (1). Recalling that $S'(t) = S(t)/L$ and $F = \kappa t/L^2$, (1) may be rewritten to yield the time, t , of complete solidification, namely,

$$t = \frac{[S(t)]^2}{4\kappa b^2} \quad (3)$$

Here, $\kappa = 6.4 \times 10^{-6}$ m²/s is the thermal diffusivity of water ice at 100 K (Hobbs, 1974) and the constant b may be obtained from a plot of $be^{b^2} \operatorname{erf}(b)$ vs. b as described in the Appendix A.

Taking $c_p = 833$ J/kg K (Petrenko and Whitworth, 1999) as the specific heat of the ice shell at $T_{\text{cold}} = 100$ K, $H = 330$ kJ/kg as the latent heat of fusion of ice at 273 K (Carslaw and Jaeger, 1959; Hobbs, 1974), $T_{\text{melting}} = 273$ K, and employing (2) as described above to determine b , returns $b = 0.438$ (Fig. 4-31 of Turcotte and Schubert, 2002; Marsh, 1989, in preparation).

Taking the initial depth of Europa's ocean to be $S(t) = 100$ km (Anderson et al., 1998; Pappalardo et al., 1999) and using these parameters in (3) returns a time for complete solidification of approximately 64 Myr. This result is reasonable under these conditions, and illustrates that in the absence of tidal heating, it is highly unlikely that any ocean would persist in Europa until the present day. The presence of a substantial liquid layer inside Europa today clearly reflects the significance of tidal heating, and to a much lesser extent, the likelihood of sulfate or chloride salts and/or mineral acids, acting as low-eutectic contaminants (Kargel, 1991a; Hogenboom et al., 1995; Kargel et al., 2000; Zolotov and Kargel, 2009; Brown and Hand, 2013).

4. Thickness of Europa's ice shell

As previously mentioned, the prevailing thickness of Europa's crust reflects a balance between local thermal and compositional regimes and outward global heat loss. The thickness of the ice shell places fundamental constraints on the proximity of water to the surface and the ease with which any subsurface melts may extrude

onto the surface. Further, it has implications for potential modes of chaos formation (e.g., see [Greenberg et al., 1999](#); [Pappalardo et al., 1999](#); [Collins et al., 2000](#); [O'Brien et al., 2002](#)). By further employing the solidification model as outlined above under the additional assumptions that Europa's ice shell is in a quasi steady-state, Stefan-style, conductive regime, where outward heat loss is balanced by heating from below due to tidal dissipation ([Fig. 1](#)), more robust constraints may be placed on the thickness of the crust. [Hussmann et al. \(2002\)](#) and [Hussmann and Spohn \(2004\)](#) explored Europa's equilibrium ice shell thickness under a variety of conditions. In contrast to those models, we do not take into account changes in viscosity in the ice shell as manifested by multiple rheological layers; we simply assume that all heating in the ice shell is due to tidal dissipation and ignore the effects of radiogenic heating from the mantle. Furthermore, in the models presented here we do not consider, in detail, Europa's thermal-orbital evolution as it relates to those of Io and Ganymede.

The total heat flow, Q_T , in Europa can be expressed as:

$$Q_T = Q_D + Q_{Rad} + Q_P + Q_S \quad (4)$$

where Q_D , Q_{Rad} , Q_P , and Q_S represent, respectively, dissipative heating, radiogenic heating, primordial heat from accretion, and heat released from solidification of the ocean. However, for the case of a quasi-static ice shell thickness, contributions from radioactivity, accretion, and crystallization of the ocean can be neglected, leaving:

$$Q_T = Q_D = Ak_c \frac{dT}{dz} \quad (5)$$

where $A = 4\pi R^2$ is Europa's surface area, k_c is the thermal conductivity of ice, and z represents depth in the shell. If as we have assumed, Q_T is mainly due to tidal dissipation, (5) may be used to determine the thickness of the icy shell. Presuming that the thickness of the shell, d , is much less than Europa's radius so that R remains relatively constant, dT becomes ΔT , while dz goes to d . Substituting these variables into (5) and rearranging allows the thickness of the icy shell to be expressed as:

$$d = \frac{k_c \Delta T 4\pi R^2}{Q_D} \quad (6)$$

where $R = 1560$ km is the mean radius of Europa, ΔT is the difference in temperature between Europa's surface ($T_{avg} = 100$ K) and the melting temperature of ice, and $k_c = 5.4$ W/m K is the thermal conductivity of water ice at 100 K ([Hobbs, 1974](#)). Note that although k_c is likely to vary in the ice shell, assuming a constant value ensures minimum insulation in the shell and thus serves to keep our results conservative. Furthermore, according to Eq. (5.6) of [Hobbs \(1974\)](#), the average thermal conductivity for an ice shell with an average surface temperature of 100 K and a basal temperature of 273 K is ~ 4 W/m K. Hence, our use of $k_c = 5.4$ W/m K to represent the thermal conductivity of the entire crust is reasonable.

Assuming a constant melting temperature of 273 K for water ice, [Hussmann and Spohn \(2004\)](#) used an expression similar to (6) to determine Europa's equilibrium ice shell thickness over time. However, the change in melting temperature of water ice with pressure must also be taken into consideration. The pressure-dependent melting temperature of ice may be expressed as $T_{melting} = 273.16 - 1.063 \times 10^{-7} P_d$ ([Kirk and Stevenson, 1987](#); [Ruiz, 2001](#)), where P_d is the pressure at a depth, d , below the surface, in Pascals. With this in mind, (6) may be rewritten as $d = \frac{k_c [(273.16 - 1.063 \times 10^{-7} \rho g d) - T_{surface}] 4\pi R^2}{Q_D}$ where $\rho = 1000$ kg/m³ is the density of water ice and $g = 1.31$ m/s² is acceleration due to gravity on Europa. Rearranging this expression returns:

$$d = \frac{(273.16K - T_{surface}) k_c 4\pi R^2}{Q_D + (1.063 \times 10^{-7} \rho g k_c 4\pi R^2)} \quad (7)$$

as the expression for ice shell thickness when the changing melting point of water ice due to pressure is considered.

It must be noted that at this stage, it is immaterial exactly where dissipation takes place on Europa, only that the dissipation budget is responsible for preventing total freezing of the ocean. Nevertheless, it has been demonstrated that a significant proportion of this budget is dissipated within the ice shell itself ([Barr and Showman, 2009](#); [Sotin et al., 2009](#)). Therefore a more detailed treatment, where the steady-state conduction equation has a term describing a heat source in the shell, is necessary. Such a treatment will be explored in Section 5. Here, however, mainly for simplicity, we assume that global heat dissipated in Europa can be accurately described by the steady-state conduction equation regardless of the exact location of the heat source.

If all energy imparted to Europa by tidal dissipation is dispersed as heat, then the amount of heat imparted to Europa may be calculated as:

$$Q_D = \frac{21}{2} \frac{k_2 (\omega R)^5 e^2}{Q_0 G} \quad (8)$$

([Roberts and Nimmo, 2008](#); [Sotin et al., 2009](#); J. Roberts, personal communication) where Q_D represents the amount of energy dissipated per unit time, k_2 is the degree-two Tidal love number, Q_0 is the dissipation factor, ω , R , and e are Europa's tidal forcing frequency, radius and orbital eccentricity, equal to 2×10^{-5} s⁻¹, ~ 1560 km, and 0.009, respectively, and G is the gravitational constant, equal to 6.67×10^{-11} m³/kg s². Assuming that $k_2 = 0.25$ and $Q_0 \sim 100$ ([Ojakangas and Stevenson, 1989](#); J. Roberts, personal communication), (8) returns $Q_D = 9.4 \times 10^{11}$ W, or, approximately 1 TW, as a representative amount of tidal energy dissipated in Europa. Substituting this value into (7) returns a value for the thickness, d , of Europa's crust, of approximately 28 km, corresponding to $T_{melting} = 269$ K at the base of the crust. Hence, the thickness of the icy shell can be thought of as being approximately 28 km per terawatt of dissipated energy. In the model of [Hussmann and Spohn \(2004\)](#), the total heat production rate in Europa was taken as the sum of heat imparted from dissipation in the ice shell and radiogenic heat sources in the mantle. Though their value for the heat production rate was somewhat lower than our calculated value for Q_D , their findings, as illustrated in Fig. 6d and f of [Hussmann and Spohn \(2004\)](#), suggest that at present, Europa's ice shell is approximately 22 km thick, which is in general agreement with our results.

In the following section, the sensitivity of d will be evaluated when Q_D and other parameters are varied.

5. Dissipative heating in the ice shell and chaos formation

As was mentioned above, when tidal heat is dissipated mainly in the ice shell ([Barr and Showman, 2009](#); [Sotin et al., 2009](#)), a slightly more involved thermal model must be considered. Under steady state conditions with a volumetric heat source, E , due to tidal dissipation, evenly distributed throughout an ice shell of thickness d , the governing heat conduction equation ((A1) and (A2)) reduces to:

$$E = -k_c \left[\frac{d^2 T}{dz^2} \right] \quad (9)$$

Here, the thermal regime is assumed at steady state, making $\partial T / \partial t = 0$ in (A2), and a heat source E (with units of W/m³) has been added to the right hand side of (A2). We also hasten to add that, although we assume that dissipation is evenly distributed in the shell, any number of models for the detailed spatial distribution of dissipative heating in the shell can be employed here. These variations can be linked to shell thickness, temperature, and/or

suspected compositional variations. Here we make the most basic assumption of even heating.

Eq. (9) can be twice integrated to yield:

$$T = -(E/2k_c)z^2 + C_1z + C_2 \quad (10)$$

Applying the usual boundary conditions of a surface temperature $T(z=0) = T_o$ and the temperature at the ice–water interface $T(z=d) = T_1$, the constants of integration can be found, namely: $C_1 = (T_1 - T_o)/d + Ed/2k_c$ and $C_2 = T_o$, which, upon substituting into (10) and algebraic manipulation, gives:

$$\frac{T(z) - T_o}{T_1 - T_o} = \left[\frac{Ed^2}{2k_c\Delta T} \right] \left[\left(\frac{z}{d} \right) - \left(\frac{z}{d} \right)^2 \right] + \left(\frac{z}{d} \right) \quad (11)$$

where $T_o = 100$ K is Europa's average surface temperature, $T_1 = 269$ K is taken to be the temperature at the base of the ice shell (i.e., at $z = d$), E is the heat source per unit volume due to tidal dissipation and $\Delta T = (T_1 - T_o)$. The remaining symbols are as previously defined. Defining non-dimensional parameters as $T' = \frac{T(z) - T_o}{T_1 - T_o}$, $E' = \frac{Ed^2}{2k_c\Delta T}$ and $z' = \frac{z}{d}$, (11) becomes, after some rearrangement:

$$T' = -E'z'^2 + z'(1 + E') \quad (12)$$

Here it is clear that in the limit of no heat source (i.e., $E' = 0$), the thermal regime goes to the expected linear conductive regime of $T' = z'$. That is, in dimensional form:

$$T(z) = T_o + \frac{(T_1 - T_o)z}{d} \quad (13)$$

Results obtained from employing (12) when $0 \leq E' \leq 10$ are shown in Fig. 3. With increasing levels of heating (i.e., increasing E'), as expected, there is an obvious maximum temperature in the mid-crust at a depth z_m , which can be found explicitly by taking the first derivative, dT'/dz' , of (12) and setting the result equal to zero, giving:

$$z_m = \frac{1}{2} \left(1 + \frac{1}{E'} \right) \quad (14)$$

where, as E' increases, z_m approaches $1/2$; this maximum is exactly at the mid-crust. For an ice shell of thickness, $d = 28$ km, the mid-crust would be 14 km below the surface. Further, for $Q = 1$ TW, as calculated in Section 4, and a radius of 1560 km, $E = (1 \times 10^{12} \text{ W} / 1.6 \times 10^{19} \text{ m}^3) = 6.3 \times 10^{-8} \text{ W/m}^3$.

The physical meaning of the parameter E' is useful to consider in more detail. From the definition above, E' can be rewritten as

$$E' = \frac{Ed/2}{k_c\Delta T/d} = \frac{Ed/2}{Q} \quad (15)$$

where the denominator is the background heat flux when there is no additional heat source (e.g., only radiogenic heating is considered), and the numerator is the magnitude of the heat source due to tidal dissipation. So the parameter E' can be thought of as a heat source whose strength is measured in multiples of Q , the background heat flow for no anomalous heating.

It is critically important to point out in these results (Fig. 3) that the level at which melting, and thus water production, takes place rises increasingly higher in the crust with increasing E . Even at the low levels of $E' = 2$ or 3, melting may extend almost 70% of the way through the crust. Although levels of dissipative heating this high may seem unreasonable, it is possible they may occur locally.

Further, it is highly unlikely that any planetary body undergoing long-term magmatism and/or volcanism can have a homogeneous crustal structure. Judging from the varied and heterogeneous nature of Europa's surface features (Pappalardo et al., 1999; Greeley et al., 1998, 2000) this is likely to be so for Europa. If so, it is also

likely that in correspondence to this inherent heterogeneity the magnitude of tidal dissipation in the crust may not be globally uniform. These possibilities were discussed in some detail by Prieto-Ballesteros and Kargel (2005), and our results (Fig. 3) are in concert with their insights.

Fig. 6b of Prieto-Ballesteros and Kargel (2005) suggests that if Europa's crust is enriched in hydrated salts, locally high thermal gradients may exist, perhaps leading to melting near the midcrust for an epsomite-rich icy shell experiencing tidal heating with magnitudes on the order of what we have considered here. Significant variations in local dissipation may therefore be due to either specific local rheological properties of the shell itself, such as the presence of low-eutectic contaminants which may locally enhance viscosity and therefore dissipation (Han and Showman, 2005; Prieto-Ballesteros and Kargel, 2005; McCarthy et al., 2007), variations in ice porosity and grain size (Nimmo and Manga, 2009), temperature variations in the ice shell (Han and Showman, 2005; Barr and Showman, 2009) or orbital influences (Hussmann et al., 2002; Hussmann and Spohn, 2004; Mitri and Showman, 2005). Such variations would cause E to vary locally and the local thermal regime would thus also vary. In areas where dissipative heating is enhanced, the crust may undergo enhanced internal heating at mid-crustal levels. This heating may destabilize the crust, especially in areas already thin, creating migrating pockets of melt water. Circumstances such as these may lead to the necessary conditions suggested by Schmidt et al. (2011) for producing chaos. Moreover, in such areas, deeper solutions from the underlying ocean may rise high in the crust, allowing for a period of high level circulation. We hasten to add however, that this overall condition of local heating and melting will certainly be transitory, and with melt production the enhanced heating will be extinguished and the crust will refreeze, perhaps adding new heterogeneity to the ice shell for another eventual cycle of elevated heating. Previous workers have also suggested that similar processes may have occurred on Europa (Hussmann et al., 2002; Hussmann and Spohn, 2004).

6. Discussion

6.1. Implications for Europa

In the case of no internal heating, the timescale found here of ~ 62 Myr for the complete crystallization of Europa's ocean is broadly similar to that of ~ 10 Myr reported by Pappalardo et al. (1998); these authors included the effect of basal solid-state convection, which would serve to hasten cooling. Our results are also in concert with the findings of Roberts and Nimmo (2008), who report a solidification time of ~ 30 Myr for a 40 km thick ocean on Enceladus.

Previous workers have shown that once Europa's ice shell becomes thicker than ~ 20 km, a basal convecting layer is likely to develop (Pappalardo et al., 1998; McKinnon, 1999; Tobie et al., 2003). The effects of solidification on a convecting ice shell on Europa were considered by Cerimele et al. (2008). Pappalardo et al. (1998) proposed that convection within such a layer, coupled with tidal dissipation in the overlying brittle portion of the ice shell, may allow a subsurface ocean to persist in Europa to the present day. A 28 km thick ice shell, as found here, is thus likely to convect (Barr and Showman, 2009). Although inclusion of a basal convective layer would alter the heat balance in the models presented, the net resistance to heat transfer in the shell would still be set by the ability of the overlying stagnant lid to conduct heat to the surface. Hence although convection would influence the thermal gradient at the base of the ice shell, the overall times and length scales considered here (i.e., solidification timescale of

the ocean and crustal thickness) would not be strongly affected. On these grounds we have not taken the effects of convection into consideration.

In terms of the ultimate sources of internal heating, radiogenic heating rates for Europa are estimated to be $\sim 0.2\text{--}0.5$ TW (Hussmann et al., 2002; Tobie et al., 2003; Hussmann and Spohn, 2004; Sotin et al., 2009), which although not insignificant, are substantially less than expected tidal heating rates. Some of the tidal heating may also be due to dissipation in the mantle, although it is not expected to exceed radiogenic heating rates (Sotin et al., 2009). Combining all of these possible sources of heating increases Q_T to 1.1–1.4 TW, which, according to (7), reduces Europa's ice shell thickness to $\sim 21\text{--}26$ km, still in general agreement with our original results for a 28 km thick shell. In spite of our model neglecting any significant contributions to the heat budget due to radiogenic heating or dissipation in the mantle, this thickness is in basic agreement with estimates of previous workers (Ojakangas and Stevenson, 1989; McKinnon, 1999; Sotin et al., 2002; Nimmo et al., 2003; Hussmann et al., 2002; Hussmann and Spohn, 2004; Mitri and Showman, 2005).

Variations in ice rheological properties and Europa's orbital evolution have been previously considered (e.g., Hussmann et al., 2002; Hussmann and Spohn, 2004). These models suggest that the amount of internal heating on Europa may have varied over time. If so, this implies that the thickness of the ice shell has also varied over time, as a function of fluctuating tidal dissipation (e.g., Hussmann and Spohn, 2004; Mitri and Showman, 2005), and also as a function of location and surface temperature. For example, in terms of the model presented here, according to (7), if the amount of tidal energy dissipated within Europa is increased to 2 TW, the average lithospheric thickness is reduced to approximately 14 km. Conversely, if it is assumed that the rate of tidal dissipation at Europa's poles, where the surface temperature is 50 K (Ojakangas and Stevenson, 1989), is equal to that at the equator, and $k_c = 10$ W/m K, representative of the thermal conductivity of water ice at 50 K (Hobbs, 1974), then Europa's shell thickness at the poles would be ~ 66 km, which corresponds to an ice melting temperature of 264 K at the base of the crust there. If however, as has been suggested by Tobie et al. (2003), the rate of tidal dissipation at the poles is 4 times greater than at the equator, then d may be as small as 16.9 km there for an ice melting temperature of 271 K at the base of the crust.

Fig. 2 shows Europa's shell thickness as a function of tidal heating rate under a variety of conditions for $T_{\text{melting}} = 269$ K. Owing to the higher thermal conductivity of water ice at 50 K relative to water ice at 100 K ($k_c = 10$ W/m K and 5.4 W/m K, respectively; Hobbs, 1974), heat may be more easily lost from the shell near the polar regions, resulting in a thicker shell there than at the equator. On the other hand, if the tidal heating rate at Europa's poles is indeed four times greater than that at the equator (Tobie et al., 2003), then the ice shell could be much thinner at the poles than it is at the equator (Fig. 2). However, imagery from the Galileo spacecraft does not seem to support this. Although local areas of chaos do indeed exist near the south pole (Riley et al., 2000, 2006), the absence of extensive chaos regions at the poles, as opposed to their common occurrence near the equator (Riley et al., 2000; Figueredo and Greeley, 2004), suggests that the ice shell is in general thicker at the poles than at the equator. Despite these inferences, it must be noted that the potential detection of water–ice plumes emanating from Europa's south polar region (Roth et al., 2014) could add credence to the possibility of a shell of heterogeneous thickness at the poles.

In a broadly similar fashion, almost any style of compositional, structural, or thickness variation in the icy crust could promote lateral changes in local rates of tidal dissipation (Prieto-Ballesteros and Kargel, 2005), which could lead to lateral variations in

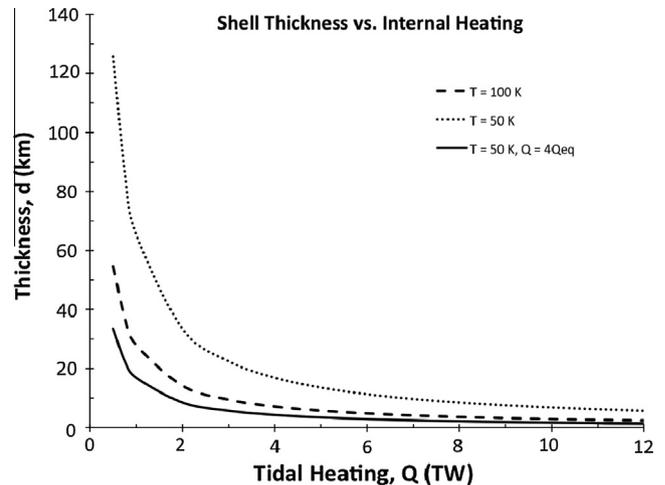


Fig. 2. Thickness of Europa's ice shell as a function of surface temperature and the magnitude of internal heating. The upper curve represents shell thickness near the poles (i.e., $T_S = 50$ K) when the heat source due to tidal dissipation is equal to that at the equator (i.e., $Q = 1$ TW). The middle curve represents shell thickness near the equator where the average surface temperature is close to 100 K. The lowermost curve represents shell thicknesses for the polar regions in the case where the heat dissipated is four times that dissipated at the equator.

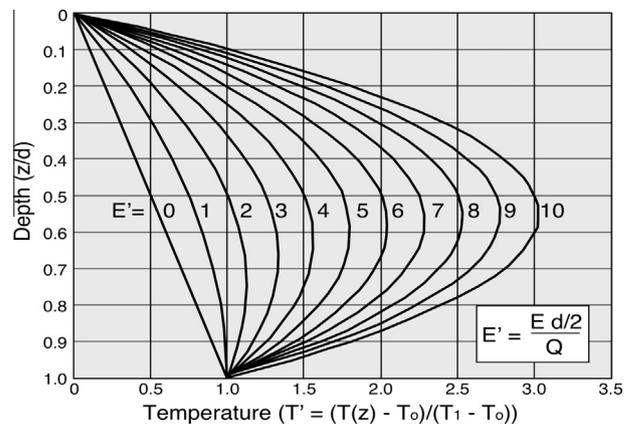


Fig. 3. Possible temperatures (dimensionless, horizontal axis) in Europa's ice shell as a function of (dimensionless) depth in the crust (vertical axis) due to locally enhanced dissipative heating (E' , dimensionless), where the magnitude of heating is given in multiples of the normal outward conductive heat flux, as marked against each curve. It is likely that $E' < 4$, but it could reach higher levels in extreme cases (e.g., local enrichments of salt). Notice that even modest amounts of anomalous heating may generate melt waters well within the crust, which in draining from the crust may destabilize the upper crust and/or siphon deeper waters upwards into the crust.

temperature and also possibly spawn sporadic melting in the crust (Fig. 3). These transitory processes would promote differentiation and local refinement of the crust (e.g., as in zone refining melting), which could introduce further structural and compositional variations, eventually leading to subtle density and strength variations and/or possibly initiating structural instabilities.

We must also note that in the model presented in Section 2, we have taken $T_{\text{melting}} = 273$ K; however since the melting temperature of water ice is pressure dependent, at a depth of 100 km on Europa ice would be expected to melt at about 259 K. Using $T_{\text{melting}} = 259$ K in (2) (with the corresponding value of $H = 2.4 \times 10^5$ J/kg; Hobbs, 1974) returns $b = 0.422$ and an ocean solidification time, $t = 69$ Myr, comparable to our original result of $t = 64$ Myr. Furthermore, although the initial condition for estimating the time to full solidification of Europa's ocean without heating was chosen

to be an ice-free, pure water ocean at time $t = 0$, this is a highly unlikely initial state. A more realistic process leading to the present conditions on Europa is an initial accreted body consisting of a heterogeneous mixture of rock and ice, perhaps with a minimum of water mainly of a transitory nature due to heating from formative impacts. The immediate pronounced effects of tidal dissipation within this *mélange* would furnish a steady source of water migrating upward in response to compensatory sinking of rock debris. Freezing of these initial waters at the surface could progressively develop an ice shell whose growing thickness would eventually provide enough insulation to allow later liquid water to collect underneath, finally attaining the present state.

In early times during ocean buildup where low temperatures and pressures prevailed, the rock of the *mélange* may certainly possess sufficient strength to resist deformation into a coherent mass of vanishing porosity. This was most likely the key to sustaining effective dissipation and water production in the past. Over time, this *mélange*–ocean interface likely migrated downward and it is to be expected that at some depth the rock component coalesced into a single, coherent layer due to compression caused by the overburden of the growing ocean. However, it is possible that this layer still contains some level of porosity in the form of fractures that reach tens of kilometers below the ocean floor (Vance et al., 2007). Hence at the hydrosphere–mantle interface, it may be possible that all rock be of only marginal strength to forego complete deformation into a consolidated mass of nil porosity. If such a boundary-layer *mélange* could persist until the present day, ongoing dissipation within this layer and circulation of hydrothermal waters through it and through any seafloor fractures beneath it, could cause occasional plumes of warm, primitive waters to ascend upward into the overlying ocean (Lowell and Dubose, 2005; Vance et al., 2007). Indeed, due to its much smaller size and thus lower internal pressures, a similar process may aid in facilitating current eruptive venting on Enceladus. On Europa, these waters could initiate diapirism in the ice shell, a process that will be considered in some detail in a subsequent investigation. Nevertheless, a more comprehensive model of tidal dissipation on Europa than we have attempted here should begin with much more general initial conditions and account for the overall size and composition of the entire satellite.

6.2. Implications of Neumann's solution for Ganymede and Callisto

6.2.1. Ocean solidification

Magnetic data returned from Galileo suggests that Ganymede and Callisto may also harbor subsurface oceans (Showman et al., 1997; Khurana et al., 1998; Kivelson et al., 1999, 2002; Zimmer et al., 2000). Detailed heat transfer calculations by previous workers (Showman et al., 1997; Spohn and Schubert, 2003) support this hypothesis, and Spohn and Schubert (2003) have shown that while an ocean is unlikely for an undifferentiated Callisto, a subsurface ocean 50–300 km thick (depending on composition, e.g., pure H₂O vs. an H₂O–NH₃ mixture) is possible for Callisto if it is partially differentiated. Assuming that the ice shells on both of these moons were also created by the progressive freezing of an ocean from the top down, Neumann's solution may be applied to place constraints on the longevity of putative oceans within these bodies and the thickness of their outer ice-I crusts.

Oceans on Ganymede and Callisto may be located at depths greater than 150 km (Zimmer et al., 2000; Kivelson et al., 2002; Spohn and Schubert, 2003; Moore and Schubert, 2003); hence the melting temperature of water ice will be depressed from 273 K in (2). $T_{cold} = 117$ K and 126 K are the average surface temperatures of Ganymede and Callisto, respectively (Moore and Schubert, 2003). These temperatures correspond to $H \cong 2.4 \times 10^5$ J/kg (Hobbs, 1974), and as before, $c_p = 833$ J/kg K

(Petrenko and Whitworth, 1999). Substituting the appropriate parameter values into (2) for each satellite returns $b \cong 0.45$ and 0.44 for Ganymede and Callisto, respectively. These b values are similar to that found for Europa in Section 2. This is no surprise as at the temperatures under consideration, heat transfer parameters for all three icy moons are nearly identical.

As for Europa, we assume that the depth of the initial ocean on each of these moons is equal to the present thickness of their icy crusts. Therefore, $S(t) = 900$ km for Ganymede, in agreement with previous estimates for the thickness of the entire ice shell (Anderson et al., 1996; Sohl et al., 2002). Estimated shell thicknesses for a partially differentiated Callisto range from 300 to 660 km (Showman and Malhotra, 1999; Anderson et al., 2001; Ruiz, 2001; Sohl et al., 2002). We therefore explore the time of complete solidification of an ocean for values of $S(t)$ within this range. Thermal diffusivities for water ice at 117 K and 126 K are 6×10^{-6} m²/s and 5×10^{-6} m²/s, respectively (Hobbs, 1974).

Substitution of these values into (3) suggests that a pure-H₂O ocean could persist within Ganymede for as much as 5.3 Gyr before freezing out. Hence such an ocean could still be liquid today, especially if it was insulated by a substantial amount of overlying ice (Spohn and Schubert, 2003), if tidal effects such as tidal blanketing (Gaeman et al., 2012) were considered, or, if it contained low-eutectic impurities such as Mg- and Na-sulfate salts (Spohn and Schubert, 2003; Vance et al., 2014). For Callisto, we find that an ocean with an initial thickness of 300 km would freeze out after ~ 740 Myr, while an ocean with an initial thickness of 660 km would persist for about 3.6 Gyr. Therefore in the absence of tidal effects at Callisto (Sohl et al., 2002; Moore et al., 2004), in order for an ocean to persist to the present day it must contain significant concentrations of low-eutectic impurities such as NH₃ or chloride or sulfate salts. These species have been previously predicted to occur on Ganymede, Callisto, and other icy satellites (Kargel, 1991a,b; Kargel et al., 1991; Hogenboom et al., 1995; Spohn and Schubert, 2003; Vance et al., 2014) and in the case of the salts, are the likely cause of the magnetic induction signals detected by the Galileo spacecraft at these moons (Khurana et al., 1998; Kivelson et al., 1999, 2002; Zimmer et al., 2000).

6.2.2. Thickness of Ganymede's ice shell

Unlike the case for Europa where tidal heating dominates over radiogenic heating, radiogenic heating is thought to contribute substantially to Ganymede's heat budget (Showman et al., 1997; Spohn and Schubert, 2003; Chen et al., 2014). Therefore in (4), Q_T for Ganymede is the sum of both a radiogenic, Q_{Rad} , and a tidal, Q_D , component. $Q_{Rad} = 3.4 \times 10^{11}$ W for Ganymede (Spohn and Schubert, 2003), and Q_D is once again calculated from (8) subject to the following parameters: $R = 2600$ km, $\omega = 1 \times 10^{-5}$ rad/s, $e = 0.0015$ (Moore and Schubert, 2003), and $Q_0 = 300$ (Showman et al., 1997). In addition, k_2 is taken to be 0.8, an average of the lowest (0.14) and highest (1.5) values considered for k_2 on Ganymede by Showman et al. (1997). Plugging these values into (8) returns $Q_D = 1.2 \times 10^{10}$ W for Ganymede, in general agreement with recent calculations for tidal heating on Ganymede (Chen et al., 2014). Combining this with the radiogenic heating rate returns $Q_T = 3.5 \times 10^{11}$ W for Ganymede. Using this value for Q_T , $k_c = 4.6$ W/m K for ice at 117 K (Hobbs, 1974), $g = 1.43$ m/s² (Kirk and Stevenson, 1987), and all other appropriate parameters for Ganymede as identified in the previous section in (7), returns $d = 149$ km for the thickness of Ganymede's outer ice-I shell, corresponding to a melting temperature of approximately 251 K at its base. These results are in agreement with the findings of Spohn and Schubert (2003) who suggested than an ocean on Ganymede may exist 145 km below the surface, and with recent findings by Vance et al. (2014), which suggest that a thermally conducting ice-I layer on Ganymede may be up to 144 km thick. In

addition, if Ganymede's subsurface ocean is 50–230 km deep, as has been previously suggested (Spohn and Schubert, 2003), our results are in concert with magnetic data from Galileo, which suggests that an ocean exists at depths between 170 and 460 km (Kivelson et al., 2002).

6.2.3. Thickness of Callisto's ice shell

Tidal heating at Callisto is expected to be insignificant (Anderson et al., 2001) as evidenced by scant signs of endogenic activity at the surface in comparison to its sister satellites (Sohl et al., 2002; Moore et al., 2004). Thus we expect that in (4), $Q_T = Q_{Rad} = 2.4 \times 10^{11}$ W at Callisto (Spohn and Schubert, 2003). Substituting this value into (7) with $R = 2400$ km (Moore and Schubert, 2003), $g = 1.24$ m/s² (Ruiz, 2001), and $k_c = 4.3$ W/m K for ice at 126 K (Hobbs, 1974) returns $d = 163$ km as the thickness of a pure water–ice shell on Callisto. This value corresponds to a melting temperature of approximately 252 K at the base of the ice shell, and is in agreement with previous work by Spohn and Schubert (2003), which suggests than an ocean on Callisto could be found at a depth of 166 km. These results are also in agreement with Zimmer et al. (2000) in that they suggest that an ocean may exist on Callisto at depths less than 200 km.

7. Conclusions

The results presented here suggest that Europa's shell is on average, 28 km thick. However, due to lower temperatures and higher thermal conductivities at the poles, the shell could be as thick as 66 km there. A substantially thinner shell at the poles, about 16.9 km thick, is possible if enhanced tidal dissipation is experienced there (Tobie et al., 2003). If Europa's entire hydrosphere is 80–170 km thick (Anderson et al., 1998), then it is indeed possible that an extensive ocean exists beneath the ice shell. Variations in shell thickness and thermo-rheological properties will also lead to significant variations in local heating, which may spawn mid-crustal melting and destabilization leading to areas of chaos at the surface.

We find that average thicknesses of the outer ice-I shells on Ganymede and Callisto are 149 km and 163 km, respectively, in agreement with the results of previous, more detailed, models that have taken heat transfer due to convection, variations in the rheology and thermal conductivity of ice, and additional tidal parameters (Ruiz, 2001; Spohn and Schubert, 2003; Moore and Schubert, 2003) into consideration.

Additionally, we find that in the absence of any internal heating, a 100 km thick ocean in Europa, in conductive equilibrium under a water–ice shell, could persist on the order of 60 Myr, while, depending on its initial thickness, an ocean in Callisto would completely crystallize in <1–4 Gyr, and an ocean in Ganymede could remain liquid for more than 5 Gyr. Significant tidal heating is necessary for a subsurface ocean to persist on Europa to the present day. Further, the maintenance of oceans in both Europa and Callisto may be further facilitated by the presence of significant amounts of low-eutectic contaminants.

The results presented here could be tested by future missions to the Jupiter system such as the European Space Agency's JUPITER ICy moons Explorer (JUICE) mission and by future missions to Europa such as the proposed Europa Clipper mission.

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Appendix A

A.1. The Stefan problem

Here we assume that initially no heat sources or any convection exist either in the solid (i.e., the ice shell) or liquid (i.e., the ocean). As such, the temperature in each domain is described by the heat equation:

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \quad (\text{A1})$$

where κ represents the thermal diffusivity of the phase under consideration. As Fig. 1 illustrates, diffusion is only in the z , or vertical, direction. Hence, (A1) may be expressed as:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \quad (\text{A2})$$

In addition, cooling is governed by the following four boundary conditions:

$$T \rightarrow T_{solid} \text{ at } z = 0 \quad (\text{i})$$

$$T \rightarrow T_{liquid} \text{ as } z \rightarrow \infty \quad (\text{ii})$$

$$T_{solid} = T_{liquid} = T_{melting} \text{ when } z = S(t) \quad (\text{iii})$$

$$q_1 - q_2 = \rho H \frac{dS}{dt} \text{ at } z = S(t) \quad (\text{iv})$$

where $T_{melting}$ is the melting temperature of the ice, ρ is the density of liquid water that changes phase at the solidification front (marked by $S(t)$), and H is the latent heat released during solidification of the liquid. The term $\rho H \frac{dS}{dt}$ represents the advance rate of the solidification front, q is the heat flux (i.e., the flow of heat per unit area) in the z -direction, and the subscripts 1 and 2 represent the solid and liquid domain, respectively (Fig. 1).

Boundary condition (iv) states that exactly at the phase-change interface, in order for the solidification front to move a distance dS , an amount of heat per unit area equal to $\rho H dS$, must be conducted away from the solid–liquid interface. From Fourier's Law of Heat Conduction, $q = -k_c \frac{dT}{dz}$, where k_c is the thermal conductivity. Hence, boundary condition (iv) may be re-written as:

$$-k_{c1} \frac{dT_1}{dz_1} + k_{c2} \frac{dT_2}{dz_2} = \rho H \frac{dS}{dt} \quad (\text{v})$$

The rate of solidification of Europa's ocean is found by solving (A2) for each medium, subject to boundary conditions (i)–(v).

A.2. Neumann's solution

At $t = 0$, a stationary infinite liquid half space at temperature T_{liquid} is considered to exist in the region $z > 0$. Suddenly at $t > 0$, the surface of the liquid half space, located at $z = 0$, is brought to a temperature $T_{cold} < T_{liquid}$. The liquid starts to solidify at a temperature $T_{melting}$. In order to determine the time taken for the entire liquid half-space to solidify, or, the time required for the entire half space to reach temperature T_{solid} , and in order to ascertain the spatial temperature distribution in the solid and liquid and the position of the solidification front at all times, a solution to (A2) must be found in both the solid and liquid mediums. Therefore, the equations to be solved are:

$$\frac{\partial T_1}{\partial t_1} = \kappa_1 \frac{\partial^2 T_1}{\partial z^2} \quad (\text{A3})$$

and

$$\frac{\partial T_2}{\partial t_2} = \kappa_2 \frac{\partial^2 T_2}{\partial z^2} \quad (\text{A4})$$

where once again, the subscripts 1 and 2 represent, respectively, the solid and liquid domains. As previously mentioned, these equations and their solutions are subject to an initial condition (I.C.) and boundary conditions (B.C.) (i)–(v), all of which may be re-stated in the following way:

$$\text{I.C. : } t \leq 0, z > 0 : T = T_{\text{liquid}}$$

$$\text{B.C. \#1 : } t > 0, z = 0 : T = T_{\text{cold}}$$

$$\text{B.C. \#2 : } t > 0, z \rightarrow \infty : T \rightarrow T_{\text{liquid}}$$

$$\text{B.C. \#3 : } t > 0, z = S(t) : T_{\text{solid}} = T_{\text{liquid}} = T_{\text{melting}}$$

$$\text{B.C. \#4 : } -k_{c1} \frac{dT_1}{dz_1} + k_{c2} \frac{dT_2}{dz_2} = \rho H \frac{dS}{dt}$$

General solutions to the heat equation take the form $T = A \operatorname{erf}\left(\frac{z}{\sqrt{4\kappa t}}\right)$ and $T = B \operatorname{erfc}\left(\frac{z}{\sqrt{4\kappa t}}\right)$ where erf is the error function and erfc is the complimentary error function, that is, $1 - \operatorname{erf}$, and A and B are constants to be determined.

Solutions satisfying (A3) and (A4) and boundary conditions #1–2 are therefore of the form:

$$T_1 = T_{\text{cold}} + A \operatorname{erf}\left(\frac{z'}{\sqrt{4F_1}}\right) \quad (\text{A5})$$

and

$$T_2 = T_{\text{liquid}} - B \operatorname{erfc}\left(\frac{z'}{\sqrt{4F_2}}\right) \quad (\text{A6})$$

Here, $z' = z/L$ and $F = \kappa t/L^2$. According to B.C. #3 at the location of the solidification front, that is at $z = S(t)$, $T_1 = T_2 = T_{\text{melting}}$. Equating (A5) and (A6) to T_{melting} and letting $z' = S'(t) = S(t)/L$, where $S'(t)$ refers to a non-dimensional form of $S(t)$, returns:

$$T_{\text{cold}} + A \operatorname{erf}\left(\frac{S'(t)}{\sqrt{4F_1}}\right) = T_{\text{liquid}} - B \operatorname{erfc}\left(\frac{S'(t)}{\sqrt{4F_2}}\right) = T_{\text{melting}} \quad (\text{A7})$$

The above expression is only true if the arguments are constants.

Therefore, one can assume that $\left(\frac{S'(t)}{\sqrt{4F_1}}\right) = b$, or

$$S'(t) = 2b\sqrt{F_1} \quad (\text{A8})$$

where b , which is a dimensionless constant of order less than or equal to unity, is a function of the boundary conditions, thermal diffusivity, specific heat, and latent heat. Expressions (A7) and (A8) may be combined to find A and B , yielding:

$$A = \frac{T_{\text{melting}} - T_{\text{cold}}}{\operatorname{erf}(b)} \quad (\text{A9})$$

and

$$B = \frac{T_{\text{liquid}} - T_{\text{melting}}}{\operatorname{erfc}(b\sqrt{\kappa_1/\kappa_2})} \quad (\text{A10})$$

where the argument of erfc in (A10) follows from (A7) and (A8), in that from (A7), $\left(\frac{S'(t)}{\sqrt{4F_2}}\right) = \left(\frac{S'(t)}{\sqrt{4\kappa_2 t/L^2}}\right)$. Using the definition of $S'(t)$ from (A8) then yields $\left(\frac{2b\sqrt{F_1}}{\sqrt{4\kappa_2 t/L^2}}\right) = \left(\frac{2b\sqrt{\kappa_1 t/L^2}}{\sqrt{4\kappa_2 t/L^2}}\right) = b\sqrt{\kappa_1/\kappa_2}$ as the argument of erfc in (A10).

For the case of Europa where liquid water transforms to ice at $z = S(t)$, $T_{\text{liquid}} = T_{\text{melting}}$, so that according to (A10), $B = 0$. (A5) and (A6) can then be expressed, respectively, as:

$$T_1 = T_{\text{cold}} + \frac{T_{\text{melting}} - T_{\text{cold}}}{\operatorname{erf}(b)} \operatorname{erf}\left(\frac{z'}{\sqrt{4F_1}}\right) \quad (\text{A11})$$

and

$$T_2 = T_{\text{liquid}} \quad (\text{A12})$$

After applying B.C. #4, setting $z' = S'(t) = 2b\sqrt{F_1}$, and employing (A9) and (A10), an expression for the constant, b , emerges:

$\frac{e^{-b^2}}{\operatorname{erf}(b)} = \frac{bH\sqrt{\pi}}{c_{p1}(T_{\text{melting}} - T_{\text{cold}})}$, which, upon rearrangement becomes:

$$be^{b^2} \operatorname{erf}(b) = \frac{c_{p1}(T_{\text{melting}} - T_{\text{cold}})}{H\sqrt{\pi}} \quad (\text{A13})$$

Here c_{p1} is the specific heat at constant pressure of the solid phase (i.e., ice).

The standard means of solving this transcendental equation is to plot the left hand side against b , and then for any value of the right hand side, which is entirely known, a corresponding value of b can be found (e.g., see Carslaw and Jaeger, 1959, p. 287). Once the numerical value of b has been obtained, it can be substituted back into (A8) and the time for solidification of the entire liquid half space, as well as the position of the solidification front as a function of time, can be extracted.

Note that as the latent heat, H , increases, the constant b decreases. This reflects the slowing of the advancement of the solidification front as the minimum value of the latent heat required to crystallize water into ice rises. For systems such as Europa, where the latent heat is large and the melting temperature and specific heat are low, so much latent heat is released and must be carried away that solidification itself is relatively slow in comparison to silicate magmas. It should be noted in passing that these general results, especially the form of (A8), have been shown to hold exceedingly well for silicate magma solidification (e.g., Marsh, 1989).

Eqs. (A8) and (A13) were employed in Section 2 as (1) and (2), respectively, to determine the solidification time of Europa's ocean.

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