# Capturing near-Earth asteroids into bounded Earth orbits using gravity assist 

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#### Abstract

In this paper, capturing Near-Earth asteroids (NEAs) into bounded orbits around the Earth is investigated. Several different potential schemes related with gravity assists are proposed. A global optimization method, the particle Swarm Optimization (PSO), is employed to obtain the minimal velocity increments for each scheme. With the optimized results, the minimum required velocity increments as well as the mission time are obtained. Results of numerical simulations also indicate that using MGAs is an efficient approach in the capturing mission. The conclusion complies with the analytical result in this paper that a NEA whose velocity relative to the Earth less than $1.8 \mathrm{~km} / \mathrm{s}$ can be captured by Earth by just one MGA. For other situations, the combination of MGAs and EGAs is better in sense of the required velocity-increments.


Keywords Asteroid • Gravity assist • Capture • Orbit

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## 1 Introduction

There are many useful resources on the NEAs (Lewis 1997; Sonter 1997) so that many terrestrial resources shortages can be supplemented by mining NEAs. So far, many asteroidrelated missions have been carried out (Zeng et al. 2014; Kawaguchi et al. 2008). Japanese Hayabusa mission is the first sample return mission in history. This mission returned a sample from asteroid Itokawa in 2003 (Kawaguchi et al. 2008). As for the asteroid capturing mission, NASA initiated Asteroid Redirect Mission (ARM) (Brophy et al. 2012; Strange et al. 2013). Two different ways to capture an asteroid were proposed in their study. The first one is to capture a whole asteroid and the second one is to bring a bulk of an asteroid to an orbit around the Moon. Recently, NASA decided to use the second scheme and they will pick up a $\sim 7 \mathrm{~m}$ diameter rock off the surface of $\mathrm{a}>100 \mathrm{~m}$ asteroid in 2020s.

As for the research works, Baoyin et al. (2010) firstly proposed the idea to capture an asteroid. In their work, a method that imposing an impulse at a NEA when it passes close to Earth so that the zero velocity curves will close in the framework of Earth-Sun circular restricted three-body problem. The best target is 2009BD for which the velocity increment is $410 \mathrm{~m} / \mathrm{s}$ in their research. Another method proposed by Hasnain et al. (2012) uses the patched conics to transfer a target NEA into the Earth Sphere of Influence (SOI), and then captures it into a bounded geocentric orbit by an impulse maneuver. They listed 23 NEAs that can be captured within 10 years. Their lowest velocity increment for capturing 2007 CB 27 is $700 \mathrm{~m} / \mathrm{s}$. Also, they suggested that the capturing velocity increment would be decreased by using the Moon flyby at a proper time. Yárnoz et al. (2013) explored an alternate method for capturing NEAs. They defined easily retrievable objects that can be transported from unperturbed
heliocentric orbits into the vicinity of the Earth at affordable costs using the invariant manifolds. They listed 12 different objects which can be retrieved with less than $500 \mathrm{~m} / \mathrm{s}$ of velocity increment. Urrutxua et al. (2015) studied how to extend the time of the capture phase of the temporarily captured asteroids with the low thrust in a high-fidelity dynamic model. The simulations indicate that the capture duration could have been prolonged for five years with relatively low velocity increment of $32 \mathrm{~m} / \mathrm{s}$ for the 2006RH120, during its latest temporary captured period.

The Gravity-assist method is an efficient way to reduce velocity increment costs and increase payloads in asteroidrelated missions. Qiao et al. (2006) studied the accessibility of NEAs via Earth Gravity Assists (EGAs). In their work, the EGA is used to reduce the launch energy and the total velocity increment for a rendezvous mission with NEAs. Chen et al. (2014) investigated the accessibility of the mainbelt asteroids using gravity assists. It is found that Mars is the most useful gravity-assist body for the main-belt asteroid missions. Yang et al. (2015) studied the low thrust transfer between a NEA and a main-belt asteroid using multiplegravity assists. As for asteroid capture missions, Gong and Li (2015a) proposed to use MGAs (MGAs) for capturing NEAs in Earth-Moon three-body restricted problem. Gong and Li (2015b) also extended their previous work (Gong and Li 2015a) to a planar restricted four-body problem. Beyond MGAs, EGAs can also be considered as a gravity-assist way and used in asteroid capture missions. Considering both MGAs and EGAs, there can be four different ways: (1) both are not used; (2) only MGAs are used; (3) only EGAs are used; (4) both are used.

In previous works, these four different ways haven't been analyzed and compared. This paper contains a numerical study on how MGAs and EGAs reduce the velocity increments required in the capturing procedure. Simulations indicate that a reasonably selected gravity-assist body would decrease the capturing velocity increment greatly. In this work, global optimizations are carried out to minimize the velocity increment required in the inbound process that captures NEAs into bounded orbits around the Earth using Particle Swarm Optimization (PSO) method for the four schemes mentioned above. This paper has two main purposes. The first one is to list the NEAs candidates for capturing as well as their minimal required velocity increments for each scheme. The second purpose of this paper is to compare the efficiency of the four different schemes. It should be noted that the outbound fuel consumption has a negligible impact on the fuel consumption of the whole mission. However, the inbound fuel consumption is always much larger than the outbound fuel consumption because of the larger mass of the spacecraft's system after capturing an asteroid. A brief discussion will be given in Sect. 3. Hence, this work only focuses on the inbound trajectories. In this case, it is easier

Fig. 1 Illustration of the impulsive gravity-assist model

to analyze the effects of various ways of gravity assists on the velocity increments.

This paper is organized in the following way. In Sect. 2, the impulse gravity-assist model is introduced and the ability of MGAs is analysis in an analytical way. The global PSO optimization methods with different gravity-assist schemes are described in Sect. 3. The efficiency of the four schemes about the gravity assist is discussed in Sect. 4. Concluding remarks are given in Sect. 5.

## 2 Dynamics

### 2.1 Impulse model of gravity assist

In a planetary mission, the flight time of spacecraft inside the influence region of gravity-assist body is very short relative to whole mission time. Hence, an assumption is always made that the position of the spacecraft keeps and the velocity has an instantaneous change during the gravity assist. Such a simplified gravity-assist model is often used in the preliminary space mission design and analysis, because of its simplicity and computational efficiency (Jiang et al. 2012; Chen et al. 2014; Yang et al. 2015).

The impulse gravity-assist model is illustrated in Fig. 1 and the following equations are satisfied in this model.
$v_{-}=v_{p}+v_{\infty-}$
$v_{+}=v_{p}+v_{\infty+}$
where $v^{-}$and $v^{+}$denote the velocity vector of asteroid before and after the gravity assist, $v_{P}$ is velocity vector of the gravity-assist planet, $v_{\infty-}$ and $v_{\infty+}$ are inbound or outbound hyperbolic excess velocity of asteroid with respect to the gravity-assist body, respectively, and $\delta$ is the turn angle. The turn angle $\delta$ is obtained as (Sergeyevsky et al. 1983)
$\delta=2 \arcsin \left(\frac{1}{1+r_{p} v_{\infty}^{2} / \mu_{p}}\right)$
where $\mu_{P}$ is gravitational constant of the gravity-assist body and $r_{p}$ is the gravity-assist radius.

To describe the geometry of three-dimensional gravity assists, a local coordinate system $o-i j k$ is defined as follows:
$i=\frac{v_{\infty-}}{v_{\infty-}}, \quad k=\frac{v_{p} \times v_{\infty-}}{\left\|v_{p} \times v_{\infty-}\right\|}, \quad j=k \times i$


Fig. 2 Illustration of the endgame with one MGA

Then, the outbound velocity can be obtained as (Chen et al. 2014; Yang et al. 2015)
$v_{\infty+}=v_{\infty}(\cos \delta i+\sin \delta \sin \varphi j+\sin \delta \cos \varphi k)$
where $\varphi$ is the direction angle.
The velocity increment induced by gravity assist is

$$
\begin{align*}
\Delta v_{G A} & =v_{\infty+}-v_{\infty-} \\
& =v_{\infty}[(\cos \delta-1) i+\sin \delta \sin \varphi j+\sin \delta \cos \varphi k] \tag{6}
\end{align*}
$$

### 2.2 Analysis of MGAs

MGAs play a very important role in capturing asteroids. In the work of Gong and Li (2015a), it was found that the required velocity increment can be very low with the help of MGAs. Meanwhile, MGAs are also used for endgame braking in the mission design for NASA Asteroid Redirect Robotic Mission concept (Strange et al. 2013). In order to look insight to the ability of MGAs for capturing asteroids, the following analysis is carried out. This analysis will also help understanding numerical optimized results with or without MGAs in Sect. 4.

The endgame with one MGA in the capture mission is illustrated in Fig. 2. At the start of the endgame, the NEA is assumed to be located at the sphere of influence (SOI) of the Earth. The NEA then moves towards the Moon for the gravity assist. In Fig. 2, the case that the NEA captured into an Earth orbit after the gravity assist is shown.

To understand the ability of the MGA, the relationship of the minimum energy of a NEA relative to the Earth after the MGA with the velocity on the SOI should be established. This purpose can be achieved with the next two steps.

Firstly, the minimum energy after MGA is connected with the inbound velocity before the MGA. The planar MGA is considered and its geometry is illustrated in Fig. 3. $v_{i n}$ is the velocity of the NEA before the MGA, $v_{m}$ is the velocity of the Moon, $v_{\infty-}$ is the inbound velocity relative to Moon, $v_{\infty+}$ is the outbound velocity relative to Moon, and $v_{\text {out }}$ is the velocity of the NEA after the MGA. To obtain the minimum energy after the MGA, the magnitude of the $v_{\text {out }}$ should be minimized. With the help of Fig. 3, the following


Fig. 3 Geometry of the planar MGA
relationship can be found:
$v_{\text {out }}=\left\{\begin{array}{l}v_{m}-v_{\infty-} \quad\left(\alpha \leq \delta_{\max }\right) \\ \sqrt{v_{m}^{2}+v_{\infty-}^{2}-2 v_{m} v_{\infty-} \cos \left(\alpha-\delta_{\max }\right)} \\ \left(\alpha>\delta_{\max }\right)\end{array}\right.$
where $\delta_{\text {max }}$ is the maximum deflection angle of NEA at minimum MGA radius $r_{\min }, \alpha$ is the angle between Moon velocity and inbound velocity of NEA. Recall Eq. (3), the $\delta_{\max }$ can be obtained by
$\delta_{\text {max }}=2 \arcsin \left(\frac{1}{1+\frac{r_{\min } \nu_{\infty}^{2}}{\mu_{m}}}\right)$
The $\alpha$ can be easily calculated with the following relationship:
$\alpha=\arccos \left(\left(v_{\infty-}^{2}+v_{m}^{2}-v_{i n}^{2}\right) /\left(2 v_{m} v_{\infty-}\right)\right)$
Besides, the magnitude of the $v_{\infty-}$ in Eq. (7) can be obtained with the magnitude of the $v_{i n}$ and $v_{m}$ by
$v_{\infty-}=\sqrt{v_{m}^{2}+v_{i n}^{2}-2 v_{m} v_{i n} \cos \theta}$
where $\theta$ is the angle between Moon velocity and NEA inbound velocity with respect to the Earth. With the calculated magnitude of the $v_{\text {out }}$, the energy of NEA after the MGA is obtained as
$E=\frac{v_{\text {out }}^{2}}{2}-\frac{2 \mu_{E}}{r_{m}}$
where $r_{m}$ is the radius of the Moon's orbit.
Secondly, the magnitude of the velocity of NEA at the Earth's SOI $V$ can be related to the magnitude of the $v_{i n}$ according to the Keplerian energy conservation, as
$v_{\text {in }}=\sqrt{(V)^{2}+2 \mu_{E} / r_{m}}$
So far, the minimum energy after the MGA can be analytically related to the velocity on the SOI with Eqs. (7)-(12).

According the analysis above, the energy of NEA after the MGA varies with its velocity $V$ at the SOI of the Earth and the angle between Moon velocity and NEA inbound velocity. The variation is illustrated in Fig. 4 and the capture condition analysis is given in Fig. 5. Because of the symmetry in location of Moon and NEA at the SOI of the Earth, only the case that $\theta$ varies from 0 to $\pi / 2$ is considered here.

It can be found from the figures above that a NEA whose velocity is less than $1.8 \mathrm{~km} / \mathrm{s}$ at the SOI of the Earth can be captured by Earth at a proper $\theta$. In addition, the highest efficiency of the MGA is achieved (i.e. the peak) when $\theta$ takes

Fig. 4 Energy after the MGA

Fig. 5 Capture condition analysis $(E<0)$ after the MGA

Fig. 6 Energy after the gravity assist with respect to $V$ when $\theta=\pi / 4$


about 40 degree. The NEA's energy change with velocity at the SOI of the Earth and $\theta$ can be more clearly illustrated in Fig. 6 and Fig. 7, respectively.

## 3 Global minimum delta-V orbits searching method

PSO is an evolutionary computation technique developed by Eberhart and Kennedy (1995). PSO is often used in global
optimization of space missions, because of its global optimization ability (Pontani and Conway 2010; Chen et al. 2014; Yang et al. 2015). PSO will be used to optimize the velocity increment of the inbound process of capturing mission in this paper. The capturing process has two steps: the first is the spacecraft transfer from the Earth parking orbit to the target asteroid and the second is to lasso the target asteroid and pull it from its original orbit to the vicinity of the Earth. The mass of the target capturing asteroid plays a deci-

Fig. 7 Energy after the gravity assist with respect to $\theta$ when $V=1 \mathrm{~km} / \mathrm{s}$


Table 1 Fuel consumption results using direct approach

| Object <br> number | Object | C 3 <br> $\left(\mathrm{~km}^{2} / \mathrm{s}^{2}\right)$ | Outbound velocity <br> increment $(\mathrm{m} / \mathrm{s})$ | Inbound velocity <br> increment $(\mathrm{m} / \mathrm{s})$ | Inbound fuel <br> consumption $(\%)$ |
| :--- | :--- | :--- | :---: | :--- | :--- |
| 1 | 2000SG344 | 2.08 | 57.61 | 941.08 | 98.52 |
| 2 | 2006RH120 | 2.09 | 565.73 | 1267.05 | 86.39 |
| 3 | 2008UA202 | 6.14 | 162.61 | 1510.85 | 95.91 |
| 4 | 2012TF79 | 0.83 | 743.29 | 1874.1 | 82.58 |
| 5 | 2014WX202 | 2.57 | 554.93 | 1967.91 | 86.8 |
| 6 | 2014QN266 | 8.01 | 124.97 | 2162.77 | 96.85 |
| 7 | 2008EA9 | 2.53 | 1091.92 | 2283.05 | 75.66 |
| 8 | 2008JL24 | 4.61 | 279.8 | 2376.41 | 93.11 |
| 9 | 2011BL45 | 2.23 | 637.97 | 2389.04 | 84.89 |
| 10 | 2012LA | 3.06 | 766.4 | 2805.41 | 82.25 |
| 11 | 2009YR | 10.68 | 427.6 | 2806.53 | 89.66 |
| 12 | 2012EP10 | 2.79 | 713.19 | 2852.56 | 83.38 |
| 13 | 2013GH66 | 6.34 | 642.59 | 2927 | 84.81 |
| 14 | 2012WR10 | 7.5 | 657.8 | 3287.48 | 84.55 |
| 15 | 2000LG6 | 2.09 | 1426.42 | 3623.41 | 69.52 |
| 16 | 2013WA44 | 9.63 | 1992.69 | 4063.36 | 59.99 |
| 17 | 2014UV210 | 0.46 | 2087.89 | 4103.41 | 58.64 |
| 18 | 2015DU | 3.77 | 2108.09 | 4287.35 | 58.34 |
| 19 | 2014YD | 2.68 | 495.78 | 5304.2 | 88.11 |
| 20 | 2014KD45 | 9.84 | 3468.41 | 5528.7 | 41.24 |

sive role in the fuel consumption of the whole capturing mission so that the fuel consumption of the second step decides the whole fuel consumption in the mission. Take picking a bulk of roughly 5 m in diameter corresponding to masses 100 tons from the candidate asteroids as an example. The computational results of escape energy C3, outbound velocity increment, inbound velocity increment, fuel consumption percentage using the PSO optimized direct approach are listed in Table 1. The fuel consumption is calculated by the following equation
$\Delta m=m_{0}\left(1-e^{\frac{-\Delta v}{1 s p \cdot g_{0}}}\right)$
where $m_{0}$ is the initial mass of the spacecraft, the specific impulse $I s p$ is assumed to be 400 s and $g_{0}$ is the standard sea level acceleration of gravity.

It can be seen from Table 1 that in 14 cases the percentage of the inbound fuel consumption is more than $80 \%$ and only in 4 cases the percentage is less than $60 \%$. Therefore, only the optimization of the inbound velocity increment is considered in this paper and it is assumed that the spacecraft and the target asteroid are connected at the initial time of the optimization.

According to the gravity-assist body, four different schemes will be investigated in this section: direct capture,

Fig. 8 Illustration of the patched conic method using one MGA

capture using only MGA, capture using only EGA and capture using both Earth and MGA.

## Case 1: direct capture

In this case, the time when the NEA starts to deviate its original orbit, $t_{0}$, and the time arriving at the Earth, $t_{f}$, are chosen as optimization variables. The sum of the velocity increments in the capturing mission is chosen as the object function:
$J=\left\|\Delta v_{0}\right\|+\left\|\Delta v_{f}\right\|$
where $\Delta v_{0}$ is the impulse executed at $t_{0}$ and $\Delta v_{f}$ is the impulse executed at $t_{f}$. With the given time of the impulses, the transfer leg can be solved by the Lambert method. Then, $\Delta v_{0}$ and $\Delta v_{f}$ are obtained by the following equations
$\Delta v_{0}=v_{s c d}-v_{a}$
$\Delta v_{f}=v_{s c a}-v_{E}$
where $v_{a}$ is velocity of NEA at $t_{0}, v_{E}$ is velocity of the Earth at $t_{f}, v_{s c d}$ and $v_{s c a}$ are velocity of the NEA after and before the impulse, respectively.

To solve the optimal problem, $x_{i}(i=1,2) \in[0,1]$ are chosen as input variables of the PSO method, so that the initial and final time can be written as
$t_{0}=t_{a d \text { min }}+d t_{a d} x_{1}$
$t_{f}=t_{0}+d t_{\text {max }} x_{2}$
where $t_{a d \text { min }}$ is the earliest NEA deviating time, $d t_{a d}$ is the window width of the NEA deviating time and $d t_{\max }$ is the maximum allowed mission time. Then, the object function is obtained by the patched conic method.

## Case 2: using one MGA

The patched conic method using one MGA is illustrated in Fig. 8. In the figure above, Lam means the transfer leg is solved by the Lambert method. The definitions of optimization variables are as follow: $t_{0}$ is the time of the joint spacecraft-NEA system departing their original orbit, $t_{\text {soi }}$ is time of the joint system arriving at the SOI of the Earth, $t_{f}$ is the time of the MGA, $\alpha$ and $\beta$ are location angles on the SOI of the Earth, $\delta_{M G A}$ and $\varphi_{M G A}$ are the direction angle and turn angle of the MGA, respectively. Three impulses are executed in the capturing mission, $\Delta v_{0}$ is the impulse at $t_{0}, \Delta v_{\text {soi }}$ is impulse at $t_{s o i}, \Delta v_{f}$ is the impulse after the MGA. Lam means that the transfer leg is solved by Lambert method.

The sum of these three impulses is chosen as the object function
$J=\left\|\Delta v_{0}\right\|+\left\|\Delta v_{s o i}\right\|+\left\|\Delta v_{f}\right\|$


Fig. 9 Illustration of the patched conic method using one EGA
where $\Delta v_{0}$ is obtained by Eq. (15) and $\Delta v_{\text {soi }}$ is velocity increment at the Earth's SOI to guidance the spacecraft-NEA system to the Moon. If the energy of the joint system relative to the Earth is greater than zero after MGA, the magnitude of $\Delta v_{f}$ is obtained by following equation
$\Delta v_{f}=\left|v_{s c m}-\sqrt{2 \mu_{E} / r_{m}}\right|$
where $v_{s c m}$ is the magnitude of velocity of the joint system at the Moon arrival, $r_{m}$ is the magnitude of the Moon position vector at gravity assist, $\mu_{E}$ is the gravitational constant of the Earth.

To solve the optimal problem, $x_{i}(i=1, \ldots, 7) \in[0,1]$ are chosen as input variables, then the object function obtained by the patched conic method is determined by the these input variables:
$t_{0}=t_{a d \text { min }}+d t_{a d} x_{1}$
$t_{\text {soi }}=t_{0}+d t_{\max } x_{2}$
$t_{f}=t_{s o i}+\left(d t_{\max }-d t_{\max } x_{2}\right) x_{3}$
$\alpha=2 \pi x_{4}$
$\beta=-0.5 \pi+\pi x_{5}$
$\varphi_{M G A}=2 \pi x_{6}$
$\delta_{M G A}=\delta_{\max } x_{7}$
where the definitions of $t_{a d \min }, d t_{a d}$ and $d t_{\text {max }}$ are the same as Case 1.

## Case 3: using one EGA

The patched conic method using one EGA is illustrated in Fig. 9. In this case, the definitions of optimization variables are as follow: $t_{0}$ is the time of the joint spacecraft-NEA system departing their original orbit, $t_{E G A}$ is the time of the joint system arrive at the EGA, $t_{D S M}$ is the time of the deep space maneuver (DSM), $t_{f}$ is the time of the joint system arrive at the Earth, $\delta_{E G A}$ and $\varphi_{E G A}$ are the direction angle and turn angle at the EGA, respectively. OP means the orbit propagation.

The overall velocity-increment of the capturing mission is chosen as the object function
$J=\left\|\Delta v_{0}\right\|+\left\|\Delta v_{D S M}\right\|+\left\|\Delta v_{f}\right\|$
where $\Delta v_{0}$ is the impulse at the NEA departure, $\Delta v_{f}$ is the impulse at the Earth arrival, and $\Delta v_{D S M}$ is the impulse of the deep space maneuver. $\Delta v_{0}$ and $\Delta v_{f}$ are decided by


Fig. 10 Illustration of the patched conic method using one EGA and one MGA

Eqs. (15) and (16). $\Delta v_{D S M}$ is the difference of the velocity of the OP leg and the velocity of the Lam leg at this point.

To solve the optimal problem, $x_{i}(i=1, \ldots, 6) \in[0,1]$ are chosen as input variables of PSO method, then the object function obtained by the patched conic method is determined by the these input variables:
$t_{0}=t_{a d \text { min }}+d t_{a d} x_{1}$
$t_{E g a}=t_{0}+d t_{\text {max }} x_{2}$
$t_{D S M}=t_{E g a}+\left(d t_{\max }-d t_{\max } x_{2}\right) x_{3}$
$t_{f}=t_{D S M}+\left(d t_{\max }-d t_{\max } x_{2}-\left(d t_{\max }-d t_{\max } x_{2}\right) x_{3}\right) x_{4}$
$\varphi_{E G A}=2 \pi x_{5}$
$\delta_{E G A}=\delta_{\max } x_{6}$
where the definitions of $t_{a d \min }, d t_{a d}$ and $d t_{\text {max }}$ are the same as previous cases.

## Case 4: using one EGA and one MGA

The patched conic method using one Earth and one MGA is illustrated in Fig. 10. The definitions of the optimization variables are as follows, $t_{0}$ is the time of the joint spacecraftNEA system departing their original orbit, $t_{E G A}$ is the time of joint system arrive at the EGA, $t_{D S M}$ is the time of the deep space maneuver, $t_{\text {Esoi }}$ is the time of the impulse at the Earth's $\mathrm{SOI}, t_{f}$ is the time of the joint system at the Moon, $\alpha$ and $\beta$ are location angles on the SOI of the Earth, $\varphi_{M G A}$ and $\delta_{M G A}$ are the direction angle and turn angle of the MGA, respectively. There are all five impulses provided by the thruster. $\Delta v_{0}$ is the impulse at $t_{0}, \Delta v_{E G A}$ is the impulse at the EGA, $\Delta v_{D S M}$ is the impulse of the deep space maneuver, $\Delta v_{\text {Esoi }}$ is the impulse at the Earth's SOI, $\Delta v_{f}$ is the impulse after the MGA.

The velocity of the asteroid after the EGA is obtained as follows
$v_{\text {after }}=v_{\text {before }}+\Delta v_{E G A}$
where the $\Delta v_{E G A}$ is determined by the two gravity-assist angles $\delta_{E G A}$ and $\varphi_{E G A}$ as shown in Fig. 1. The velocityincrement of the capturing mission is chosen as the object function
$J=\left\|\Delta v_{0}\right\|+\left\|\Delta v_{D S M}\right\|+\left\|\Delta v_{E s o i}\right\|+\left\|\Delta v_{f}\right\|$
To solve the optimal problem, $x_{i}(i=1, \ldots, 11) \in[0,1]$ are chosen as the input variables of the PSO method, then
the object function obtained by the patched conic method is determined by the these input variables:
$t_{0}=t_{a d \text { min }}+d t_{a d} x_{1}$
$t_{E g a}=t_{0}+d t_{\max } x_{2}$
$t_{D S M}=t_{E g a}+\left(d t_{\max }-\left(t_{E g a}-t_{0}\right)\right) x_{3}$
$t_{\text {Esoi }}=t_{D S M}+\left(d t_{\text {max }}-\left(t_{D S M}-t_{0}\right)\right) x_{4}$
$t_{f}=t_{E s o i}+\left(d t_{\max }-\left(t_{E s o i}-t_{0}\right)\right) x_{5}$
$\alpha=2 \pi x_{6}$
$\beta=-0.5 \pi+\pi x_{7}$
$\varphi_{E G A}=2 \pi x_{8}$
$\delta_{E G A}=\delta_{\max } x_{9}$
$\varphi_{M G A}=2 \pi x_{10}$
$\delta_{M G A}=\delta_{\max } x_{11}$

## 4 Numerical simulation and discussion

### 4.1 Numerical simulation results

In the numerical simulation, the orbital elements of the asteroids are all from the Near-Earth Objects Dynamic Site. ${ }^{1}$ For the computational efficiency, only the NEAs whose inclination less than 7 deg and eccentricity less than 0.2 are selected. Firstly, 20 best candidate NEAs that can be captured in the direct way are selected. Then, the optimum velocity increments for these 20 candidates in four different ways are calculated. Given the associated parameter, the optimal variables can be achieved by the PSO optimizer using the method presented in the previous section.

The constraint parameters are given as follows
$T_{a d \text { min }}=1$ st Jan, 2025
$d t_{L d}=5$ years
$d t_{\max }=1000$ days
The optimal results of the 20 candidates NEAs in the different four cases are in Tables 2, 3 and 4.

An example of capturing the 2000SG344 using EGA is illustrated in Fig. 11.

### 4.2 Discussion

For the comparison, the total velocity increment and mission time of the different cases are illustrated in Figs. 12 and 13.

Within specified time constraints, it can be seen clearly from Fig. 12 that the minimum capturing velocity increment in Case 1 is $700 \mathrm{~m} / \mathrm{s}$ for the candidate NEA 2006RH120, the maximum velocity increment is $2226.14 \mathrm{~m} / \mathrm{s}$ for NEA 2013GH66, such a high velocity increment requirement is beyond the current thruster capability. Therefore, a mission

[^1]Table 2 The optimal result of PSO method without gravity-assist

| Object <br> number | Object | Delta-V at NEA <br> departure $(\mathrm{m} / \mathrm{s})$ | Delta-V at Earth <br> arrival $(\mathrm{m} / \mathrm{s})$ | Total mission <br> time (day) | Total delta-V <br> $(\mathrm{m} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 1 | 2006RH120 | 157.74 | 542.35 | 197.87 | 700.08 |
| 2 | 2000SG344 | 232.36 | 583.73 | 204.15 | 816.09 |
| 3 | 2012TF79 | 631.52 | 413.55 | 165.32 | 1045.07 |
| 4 | 2008UA202 | 440.07 | 870.37 | 253.21 | 1310.44 |
| 5 | 2008EA9 | 270.95 | 1220.87 | 356.26 | 1491.82 |
| 6 | 2014QN266 | 556.95 | 1136.05 | 264.77 | 1693.00 |
| 7 | 2014WX202 | 42.59 | 1659.37 | 340.54 | 1701.96 |
| 8 | 2011BL45 | 1662.70 | 88.37 | 300.71 | 1751.07 |
| 9 | 2012WR10 | 329.55 | 1425.65 | 282.13 | 1755.20 |
| 10 | 2013WA44 | 777.73 | 1234.69 | 323.85 | 2012.42 |
| 11 | 2014UV210 | 1843.20 | 171.30 | 233.74 | 2014.50 |
| 12 | 2012LA | 374.61 | 1661.92 | 280.99 | 2036.54 |
| 13 | 2009YR | 543.87 | 1502.40 | 203.48 | 2046.27 |
| 14 | 2012EP10 | 970.31 | 1085.91 | 246.00 | 2056.22 |
| 15 | 2014KD45 | 812.88 | 1247.41 | 327.89 | 2060.29 |
| 16 | 2008JL24 | 762.84 | 1303.84 | 239.03 | 2066.68 |
| 17 | 2000LG6 | 372.44 | 1788.18 | 288.65 | 2160.62 |
| 18 | 2014YD | 1498.49 | 676.16 | 303.47 | 2174.65 |
| 19 | 2015DU | 1478.71 | 700.56 | 323.78 | 2179.27 |
| 20 | 2013GH66 | 1218.51 | 1007.63 | 244.14 | 2226.14 |

Table 3 The optimal result of PSO method using MGA

| Object <br> number | Object | Delta-V at NEA <br> departure $(\mathrm{m} / \mathrm{s})$ | Delta-V at Earth <br> SOI $(\mathrm{m} / \mathrm{s})$ | Delta-V after <br> MGA $(\mathrm{m} / \mathrm{s})$ | Total mission <br> time $($ day $)$ | Total delta-V <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :--- | :---: | :---: | :--- | :--- | :---: |
| 1 | 2006RH120 | 457.29 | 0.00 | 0.00 | 125.05 | 457.29 |
| 2 | 2000SG344 | 78.75 | 0.00 | 0.00 | 236.30 | 78.75 |
| 3 | 2012TF79 | 870.19 | 0.00 | 0.00 | 302.53 | 870.19 |
| 4 | 2008UA202 | 387.84 | 0.00 | 0.00 | 261.67 | 387.84 |
| 5 | 2008EA9 | 256.59 | 12.88 | 0.00 | 365.29 | 269.47 |
| 6 | 2014QN266 | 469.31 | 0.00 | 0.00 | 312.56 | 469.31 |
| 7 | 2014WX202 | 449.43 | 0.00 | 0.00 | 345.72 | 449.43 |
| 8 | 2011BL45 | 262.58 | 0.00 | 0.00 | 302.33 | 262.58 |
| 9 | 2012WR10 | 325.50 | 0.00 | 0.00 | 294.65 | 325.50 |
| 10 | 2013WA44 | 611.53 | 0.07 | 0.00 | 338.81 | 611.60 |
| 11 | 2014UV210 | 527.14 | 338.87 | 0.00 | 339.24 | 866.01 |
| 12 | 2012LA | 610.64 | 503.62 | 0.00 | 240.08 | 1114.26 |
| 13 | 2009YR | 483.94 | 285.62 | 0.00 | 251.83 | 769.56 |
| 14 | 2012EP10 | 328.10 | 397.01 | 0.00 | 318.00 | 725.12 |
| 15 | 2014KD45 | 807.00 | 388.26 | 0.00 | 342.15 | 1195.26 |
| 16 | 2008JL24 | 334.16 | 366.85 | 0.00 | 248.94 | 701.01 |
| 17 | 2000LG6 | 339.65 | 0.00 | 0.00 | 290.38 | 339.65 |
| 18 | 2014YD | 1365.75 | 0.00 | 0.00 | 213.80 | 1365.75 |
| 19 | 2015DU | 1107.52 | 0.00 | 0.00 | 330.03 | 1107.52 |
| 20 | 2013GH66 | 796.12 | 0.00 | 0.00 | 140.94 | 796.12 |

without a gravity assist is infeasible now. As for Case 2, MGA greatly decrease the capturing velocity increment. The minimum capturing velocity increment is $78.75 \mathrm{~m} / \mathrm{s}$ for the

NEA 2000SG344 and the maximum capturing velocity increment is $1365.75 \mathrm{~m} / \mathrm{s}$ for the NEA 2014YD. As for Case 3 , the minimum capturing velocity increment is $462.9 \mathrm{~m} / \mathrm{s}$ for

Table 4 The optimal result of PSO method using EGA

| Object <br> number | Object | Delta-V at NEA <br> $(\mathrm{m} / \mathrm{s})$ | Delta-V at deep <br> space $(\mathrm{m} / \mathrm{s})$ | Delta-V at Earth <br> arrival $(\mathrm{m} / \mathrm{s})$ | Total mission <br> time $($ day $)$ | Total delta-V <br> $(\mathrm{m} / \mathrm{s})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2006RH120 | 158.66 | 207.90 | 96.34 | 851.80 | 462.90 |
| 2 | 2000SG344 | 229.42 | 156.68 | 122.56 | 680.71 | 508.66 |
| 3 | 2012TF79 | 476.77 | 147.86 | 133.32 | 582.56 | 757.95 |
| 4 | 2008UA202 | 434.15 | 226.47 | 200.27 | 709.85 | 860.89 |
| 5 | 2008EA9 | 240.87 | 341.84 | 282.49 | 819.56 | 865.20 |
| 6 | 2014QN266 | 544.46 | 291.67 | 275.49 | 721.52 | 1111.62 |
| 7 | 2014WX202 | 41.62 | 454.19 | 371.10 | 817.66 | 866.91 |
| 8 | 2011BL45 | 524.59 | 415.56 | 340.65 | 810.27 | 1280.81 |
| 9 | 2012WR10 | 340.05 | 376.40 | 322.83 | 737.25 | 1039.28 |
| 10 | 2013WA44 | 642.14 | 375.26 | 318.99 | 801.23 | 1336.39 |
| 11 | 2014UV210 | 1847.58 | 54.86 | 32.49 | 733.24 | 1934.93 |
| 12 | 2012LA | 374.77 | 436.07 | 380.89 | 741.08 | 1191.72 |
| 13 | 2009YR | 495.25 | 428.43 | 348.32 | 679.52 | 1272.00 |
| 14 | 2012EP10 | 958.03 | 281.65 | 260.01 | 692.47 | 1499.69 |
| 15 | 2014KD45 | 821.04 | 323.75 | 286.69 | 779.20 | 1431.48 |
| 16 | 2008JL24 | 192.25 | 544.00 | 558.34 | 700.03 | 1294.60 |
| 17 | 2000LG6 | 344.34 | 656.68 | 302.38 | 913.47 | 1303.39 |
| 18 | 2014YD | 1477.87 | 177.05 | 158.78 | 751.82 | 1813.70 |
| 19 | 2015DU | 1081.40 | 336.61 | 318.32 | 791.60 | 1736.34 |
| 20 | 2013GH66 | 117.42 | 569.16 | 486.58 | 738.58 | 1173.16 |



Fig. 11 Transfer trajectory of the 2000SG344 NEA in heliocentric coordinate system
the NEA 2006RH120 and the maximum capturing velocity increment is $1934.93 \mathrm{~m} / \mathrm{s}$ for the NEA 2014UV210. As for Case 4, the minimum capturing velocity increment is $92.65 \mathrm{~m} / \mathrm{s}$ for the NEA 2000SG344 and the maximum capturing velocity increment is $1630.48 \mathrm{~m} / \mathrm{s}$ for the NEA 2014YD. In general, the methods using MGA and EGA are more efficient than the others. For example, the capturing velocity increment for the 2000LG6 NEA using MGA can save $1821 \mathrm{~m} / \mathrm{s}$ compared with cases where gravity assists are not used.

Form the 5th column of Table 3 and 6th column of Table 5, a conclusion can be drew that the NEAs can be captured by the Earth after just one MGA. The conclusion is consistent with previous analysis. By comparing the 4th column of Table 3 and the 5th column of Table 5, it can be seen that the EGA decrease the velocity-increment at the SOI of the Earth greatly. However, such a method will increase the mission time. It can be seen from Fig. 13 that the cases using EGA have the mission time increased an average of 460 days.

It should be pointed out that all the results given above are based on the fact that only the inbound trajectories are considered. For a whole mission of capturing an asteroid, the fuel consumption of the outbound trajectories may play an important role and should be further analyzed. Besides, the whole or part of the escape energy is provided by the launcher so that the escape energy should not exceed the launcher's capability and the corresponding propellant budget should also be checked in actual missions.

## 5 Conclusion

In this paper, four different cases of capturing NEAs according to the gravity assists are studied. For each case, the way of using the PSO method to obtain the minimal velocity increments in the inbound process is proposed. Through the

Fig. 12 Total delta-V for different cases


Fig. 13 Total mission time for different cases


Table 5 the optimal result of PSO method using EGA and MGA

| Object <br> number | Object | Delta-V at NEA <br> $(\mathrm{m} / \mathrm{s})$ | Delta-V at deep <br> space $(\mathrm{m} / \mathrm{s})$ | Delta-V at Earth <br> SOI $(\mathrm{m} / \mathrm{s})$ | Delta-V after <br> MGA $(\mathrm{m} / \mathrm{s})$ | Total mission <br> time $($ day $)$ | Total delta-V <br> $(\mathrm{m} / \mathrm{s})$ |
| :--- | :--- | :--- | :---: | :--- | :--- | :---: | :---: |
| 1 | 2006RH120 | 141.72 | 264.81 | 25.30 | 0.00 | 645.85 | 431.83 |
| 2 | 2000SG344 | 91.97 | 0.68 | 0.00 | 0.00 | 954.69 | 92.65 |
| 3 | 2012TF79 | 497.04 | 178.23 | 0.07 | 0.00 | 642.41 | 675.34 |
| 4 | 2008UA202 | 107.01 | 234.07 | 0.00 | 0.00 | 728.74 | 341.08 |
| 5 | 2008EA9 | 287.22 | 0.14 | 0.00 | 0.00 | 736.30 | 287.37 |
| 6 | 2014QN266 | 519.09 | 0.37 | 0.00 | 0.00 | 644.28 | 519.46 |
| 7 | 2014WX202 | 624.06 | 1.77 | 0.00 | 0.00 | 710.08 | 625.83 |
| 8 | 2011BL45 | 286.74 | 82.37 | 1.35 | 0.00 | 640.90 | 371.56 |
| 9 | 2012WR10 | 310.78 | 1.24 | 0.00 | 0.00 | 683.93 | 312.01 |
| 10 | 2013WA44 | 757.49 | 6.95 | 0.00 | 0.00 | 682.48 | 764.43 |
| 11 | 2014UV210 | 559.07 | 0.34 | 338.28 | 0.00 | 680.14 | 897.69 |
| 12 | 2012LA | 427.95 | 80.46 | 0.01 | 0.00 | 830.20 | 508.41 |
| 13 | 2009YR | 647.53 | 50.38 | 0.00 | 0.00 | 578.46 | 697.91 |
| 14 | 2012EP10 | 925.64 | 0.39 | 0.00 | 0.00 | 639.40 | 926.03 |
| 15 | 2014KD45 | 879.76 | 22.76 | 0.00 | 0.00 | 831.23 | 902.52 |
| 16 | 2008JL24 | 539.26 | 137.67 | 0.13 | 0.00 | 1001.27 | 677.06 |
| 17 | 2000LG6 | 334.99 | 1.69 | 0.00 | 0.00 | 658.13 | 336.68 |
| 18 | 2014YD | 1450.43 | 180.05 | 0.00 | 0.00 | 881.86 | 1630.48 |
| 19 | 2015DU | 844.98 | 10.33 | 0.16 | 0.00 | 714.09 | 855.47 |
| 20 | 2013GH66 | 159.58 | 0.41 | 361.25 | 0.00 | 448.48 | 521.24 |

optimization, the minimal velocity-increment requirements for capturing the NEAs between 2025 and 2030 are presented for the selected targets. These results can offer references for preliminary mission designs in the future. The conclusions on the optimization of capturing NEAs to the bounded Earth orbit are summarized as follows

1. The results of PSO optimization method with MGA and the EGA indicate that both types of gravity assists are similar in reducing velocity increments. However, the latter takes more time than the former. The NEA candidate list can be enlarged by using MGAs or EGAs.
2. The minimum delta-V for capture NEA is $78.75 \mathrm{~m} / \mathrm{s}$ with MGA and the target is 2000SG344.
3. According to the analysis of MGAs, the NEAs whose velocity at the SOI of Earth less than $1.8 \mathrm{~km} / \mathrm{s}$ can be captured by Earth after just one MGA. If its velocity exceeds $1.8 \mathrm{~km} / \mathrm{s}$, EGAs can be used to decrease this value to be less than $1.8 \mathrm{~km} / \mathrm{s}$. Then, it can be captured by using MGA again. The simulation results comply with the analysis in Sect. 2.

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## References

Baoyin, H.X., Chen, Y., Li, J.F.: Capturing near Earth objects. Res. Astron. Astrophys. 10(6), 587 (2010)
Brophy, J., Culick, F., Friedman, L., Allen, C., Baughman, D., Bellerose, J., Yeomans, D.: Asteroid retrieval feasibility study. Keck Institute for Space Studies, Califonia Institute of Technology, Jet Propulsion Laboratory (2012)
Chen, Y., Baoyin, H., Li, J.: Accessibility of main-belt asteroids via gravity assists. J. Guid. Control Dyn. 37(2), 623-632 (2014)

Eberhart, R.C., Kennedy, J.: A new optimizer using particle swarm theory. In: Proceedings of the Sixth International Symposium on Micro Machine and Human Science, vol. 1, pp. 39-43 (1995)
Gong, S., Li, J.: Asteroid capture using lunar flyby. Adv. Space Res. (2015a)
Gong, S., Li, J.: Planetary capture and escape in the planar four-body problem. Astrophys. Space Sci. 357(2), 1-10 (2015b)
Hasnain, Z., Lamb, C.A., Ross, S.D.: Capturing near-Earth asteroids around Earth. Acta Astronaut. 81(2), 523-531 (2012)
Jiang, F., Baoyin, H., Li, J.: Practical techniques for low-thrust trajectory optimization with homotopic approach. J. Guid. Control Dyn. 35(1), 245-258 (2012)
Kawaguchi, J.I., Fujiwara, A., Uesugi, T.: Hayabusa-its technology and science accomplishment summary and Hayabusa-2. Acta Astronaut. 62(10), 639-647 (2008)
Lewis, J.S.: Resources of the asteroids. J. Br. Interplanet. Soc. 50(2), 51-58 (1997)
Pontani, M., Conway, B.A.: Particle swarm optimization applied to space trajectories. J. Guid. Control Dyn. 33(5), 1429-1441 (2010)

Qiao, D., Cui, H., Cui, P.: Evaluating accessibility of near-Earth asteroids via EGAs. J. Guid. Control Dyn. 29(2), 502-505 (2006)
Sergeyevsky, A.B., Snyder, G.C., Cunniff, R.A.: Interplanetary Mission Design Handbook, vol. I, Part 2. JPL Publication 82-43, Pasadena (1983)
Sonter, M.J.: The technical and economic feasibility of mining the near-Earth asteroids. Acta Astronaut. 41(4), 637-647 (1997)
Strange, N., Landau, T.M., Lantoine, G., Lam, T., McGuire, M., Burke, L., Dankanich, J.: Overview of mission design for NASA asteroid redirect robotic mission concept. In: 33rd International Electric Propulsion Conference, The George Washington University, Washington, DC (2013)
Urrutxua, H., Scheeres, D.J., Bombardelli, C., Gonzalo, J.L., Peláez, J.: Temporarily captured asteroids as a pathway to affordable asteroid retrieval missions. J. Guid. Control Dyn. (2015)
Zeng, X., Gong, S., Li, J.: Fast solar sail rendezvous mission to near Earth asteroids. Acta Astronaut. 105(1), 40-56 (2014)
Yárnoz, D.G., Sanchez, J.P., McInnes, C.R.: Easily retrievable objects among the NEO population. Celest. Mech. Dyn. Astron. 116(4), 367-388 (2013)
Yang, H., Li, J., Baoyin, H.: Low-cost transfer between asteroids with distant orbits using multiple gravity assists. Adv. Space Res. (2015)


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[^1]:    ${ }^{1}$ http://newton.dm.unipi.it/neodys/ [retrieved on 25 March 2015].

