

Name: _____

Earth 110 – Exploration of the Solar System
Assignment 1: Celestial Motions and Forces

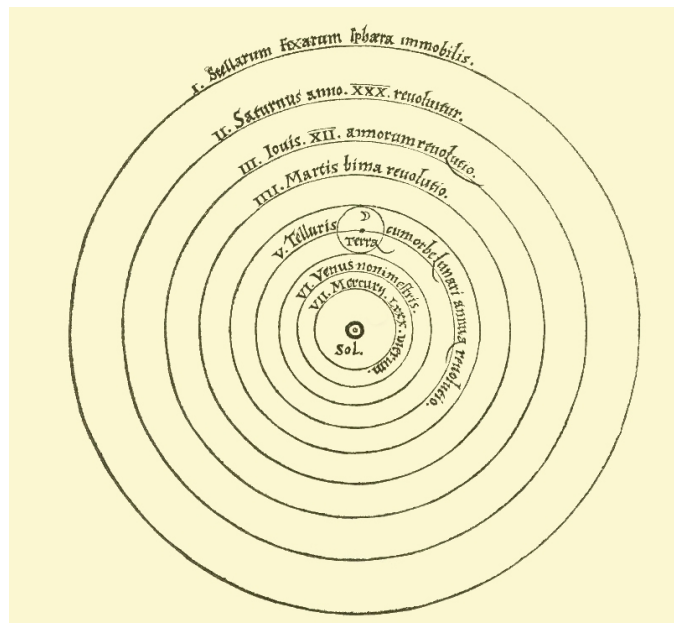
Due on Tuesday, Jan. 19, 2016

Why are celestial motions and forces important? They explain the world around us. They tell us why and how planets orbit stars, and what happens when two objects interact with each other. They also explain the most important force between two objects in space: *gravity*. Gravity affects us here on Earth: it's why we land when we jump, it keeps our atmosphere from escaping, and it's what makes it so hard for rockets and spacecraft to leave the surface (among many other things). Gravity is also the reason why we would feel lighter on a smaller object like the Moon and why we would feel heavy on a larger planet. These motions and forces boil down to a few key variables: mass, distance, velocity, and acceleration.

This assignment is meant to give you a little bit of a mathematical insight into why planets move and how they move. While we use Mercury and Venus as examples, it is beneficial to think what this means for Earth, and consequently for you. In particular, the shape of Earth's orbit, its speed, its distance from the Sun, and its interactions with the Moon, Sun, and other planets. After all, whatever happens to Earth also happens to us.

Material presented here is related to Chapters 1, 3, and 4 of the assigned textbook. This book, a really excellent resource, describes astronomical concepts in a very approachable way. Reading this book will be beneficial for doing well in this course. Also, if you are having trouble with the assignment or any concepts presented in class, we recommend consulting the textbook first.

For all calculations, make sure to show your work, include units, and clearly mark your answers (e.g., box, star, arrow)! Working in pencil is HIGHLY recommended.



The Copernican Model of our Solar System

In order to understand celestial motions and forces, we must first get a good handle on the scale of the solar system and “nearby” space. When talking about solar system distances, we use the term **astronomical unit (AU)**. The Earth is 1 AU from the Sun, equaling about 150 million kilometers or 93 million miles.

If you were traveling at 70 miles per hour, how long would it take you to travel 1 AU? How long would it take you to travel to Neptune (30 AU)? Express in years. (4 pts)

Light travels at about 3×10^8 meters per second. How long does it take for light from the Sun to reach Earth (express in *minutes*, make sure to check your units!)? (3 pts)

Extra credit: On a separate sheet of paper, calculate how long it would take radio communications to reach Mars and return to Earth. You may have to look in the book for some values. How does this compare with radio communications to the Moon and back? What would this mean for future human exploration to Mars? **Show all your work.**

Although light travels very fast, it still takes time to reach us. When astronomers say a star is 40 **light years** away from us, it means that it took light 40 years to travel from that star to our eyes (so a light year is a unit of *distance*, not time). It also means that we now see the star as it looked 40 years ago. This notion is applied to all stars (even the ones billions of light years away) in the universe because everything is so far away from us. So when you look up at the night sky, you’re really looking into the past!

What is the actual distance of a light year (Hint: distance = velocity * time)? Express in km. (2 pts)

How does the actual light year distance compare to that of an AU? Express as “1 light year distance equals X AUs”. (3 pts)

The nearest star to us, Proxima Centauri, is 4.2 light years away. To escape the influence of the Sun, our rockets need to travel at 36,500 miles per hour (56,741 km per hour). At this rate, how long would it take you to get to Proxima Centauri? Express in years. Comment on some of the issues humans would encounter when spending this amount of time in space. (4 pts)

Kepler’s Laws of Planetary Motion

Johannes Kepler devised three laws to explain the motion of the planets. His first law states that planets orbit a star in ellipses rather than perfect circles. The degree to which a planet’s orbit varies from circular is called *eccentricity*. An eccentricity of zero is perfectly circular, while an eccentricity of one is a straight line. The eccentricity (e) of Mercury measures 0.2056 while that of Venus measures 0.0068. Sketch what these orbits would look like (make sure to label them). (4 pts)

Kepler's second law deals with orbital areas: planets sweep out equal areas in equal times. A planet's distance from the Sun varies because its orbit is an ellipse. The closest point to the Sun is called the *perihelion* and its farthest point is called the *aphelion*. In order for this law to be true, it means that planets travel faster when closer to the Sun and slower when farther away. Isaac Newton explained this velocity change as a consequence of the Sun's gravity (see Newton's Laws of Motion).

Calculate the perihelion and aphelion distances for Mercury (semi major axis, a , is 0.39 AU) using the equations **perihelion distance** = $a(1-e)$ and **aphelion distance** = $a(1+e)$. How do these values compare to those of Venus ($a = 0.72$ AU)? What does this mean for Venus' velocity throughout its orbit? (5 pts)

Lastly, Kepler's third law explains that the farther a planet is from the Sun, the longer it takes for it to complete one orbit. This is expressed as $p^2 = a^3$, where p is the orbital period (in years) and a is the semi major axis (in AU).

We can manipulate this law to calculate the average orbital speed (v) of a planet, where

$v = \frac{2\pi}{\sqrt{a}}$ in AU per year. Calculate v for Mercury and Venus and convert to miles per hour. Why are they different? (5 pts)

Extra credit: On a separate piece of paper, derive the above velocity equation from Kepler's third law. Include a sketch of where your variables/values come from. **Show all your work.**

Newton's Laws of Motion

Isaac Newton developed three laws of motion that connected Earth with the heavens. His first law, an object moves at constant velocity if there is no net force acting upon it, explains how spacecraft don't need fuel to keep moving in space and why a ball slows down after it is thrown (air resistance).

Newton's second law explains what happens when a force is present. A force (F) is a rate of change of momentum, or $F = m * a$, where m is mass and a is acceleration. Think of it this way: when you're driving in a car and you hit the gas pedal, you are changing the acceleration of the car (it is increasing), thus changing the force (increasing) and its momentum since the mass doesn't change (although mass gets tricky when you approach the speed of light – shout out for Einstein).

Newton's second law also explains why planets orbit stars. A planet orbiting a star has an acceleration in the direction of the star, and gravity is the force that is causing the acceleration (see Chapter 4 for a more thorough explanation).

Newton's third law tells us that for any force, there is always an equal and opposite reaction force. This is particularly important for astronomy because it tells us that objects influence each other through gravity. When you skydive, your gravitational force is actually influencing the Earth. However, your force is much, much smaller compared to Earth's, so your influence on Earth is not noticeable.

Newton's laws follow the law of conservation of momentum: in the absence of external forces, the total momentum of interacting bodies does not change, it is *conserved*. An object can only gain or lose momentum if the momentum of some other object changes by exactly the opposite amount.

Apollo 11 launched from Cape Canaveral on a Saturn V rocket and traveled to the Moon. Explain how Newton's three laws come into play during certain times in this launch and explain how momentum is conserved. (Hint: Start with rocket ignition. If you're having trouble, check out Section 4.3 in the textbook). (10 pts)

Universal Law of Gravitation

The most important force that governs the planets and stars is gravity. Newton came up with an expression for the force of gravitational acceleration between two objects (F_g):

$F_g = \frac{GMm}{r^2}$ where G is the gravitational constant ($6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2}$), M and m are the masses of the two objects, and r is the distance between the centers of the two objects.

What happens to F_g when the distance between the two objects increases by a factor of 2? A factor of 4? What about when $M = m$? When $M = 4m$? Which variable plays the greatest role in determining gravitational forces? (10 pts)

Galileo discovered that the acceleration of a falling object is independent of its mass m . Use the universal law of gravitation and Newton's second law to show this (Hint: set the two forces equal to each other). Show your work. (6 pts)

Apollo 15 astronauts tested this hypothesis while they were on the Moon: http://youtu.be/5C5_dOEyAfk. What would happen if you performed this experiment on Earth? Why? (4 pts)